CATalytic Insurance: the case of natural disasters.*

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Abstract

Why should developing countries be interested in catastrophe (CAT) insurance? Abstracting from risk aversion or hedging motives, we show that insurance may have a catalytic role on external finance. Such effect is particularly strong in those low to middle income countries that face financial constraints when hit by a shock or in its anticipation. Insurance makes defaults less likely, thereby relaxing the country's borrowing constraint, and enhancing its access to capital markets. The presence of multilateral lenders that explicitly or implicitly provide inexpensive reconstruction funds in the aftermath of a natural disaster weakens but does not eliminate the demand for catalytic insurance.

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1 Introduction

All countries are exposed and vulnerable to natural disasters.[1] Exposure is mostly determined by geographic characteristics; vulnerability by policies and economic development. Indeed, richer countries have more resources to finance risk mitigation activities in preparation for, or reconstruction activities in the aftermath of, a shock. This may well explain why the cost (both in terms of human lives and economic costs) of the 2010 Haiti was much larger than that of the same year's Chile earthquake despite the fact that the latter was much stronger. Not to say what would have happened had the 2011 Japan's earthquake hit a less developed country.

If countries differ in their exposure and vulnerability to natural disasters, should they deal with them differently? What are the pros and cons of purchasing catastrophe insurance versus doing nothing and/or relying on ex-post lending facilities such as the ones offered by international donors and financial institutions? To answer these important questions, this paper presents a simple model that sheds new light on the trade-offs that countries face in making such difficult choices.

It is well known that market insurance, if reasonably priced, is the most effective way for risk adverse agents to cope with large and rare events.[2] This is true for individuals as well as for sovereigns. Indeed, there are a few examples of countries that insured themselves against natural catastrophes.[3] In May 2006, the Mexican government issued a US 160 million dollar parametric catastrophe bond to finance rescue and rebuilding in the case a major earthquake hits some densely populated areas of the country;[4] in 2007, a pool of Caribbean countries developed a Caribbean Catastrophe Risk Insurance Facility (CCRICF) that facilitates their access to the insurance market. However, catastrophe insurance is expensive and, even in successful cases such as the two just mentioned, the cost of coverage turned out to be a multiple of the actuarially fair price (around three times in either case).[5]

Why is insurance so costly? Several reasons are invoked, including supply-side constraints induced by either agency costs or adverse selection, problems of information opacity of tail events, coordination failures, taxes, and oligopolistic practices.[6] While the securitization of catastrophic risk through the issuance of catastrophe bonds may in the future induce greater market discipline, until now it has fallen short of reducing the costs of insurance to actuarially fair levels.[7]

If insurance is so expensive, why would countries still buy it? One reason could be risk aversion; another could be the presence of concavities in the production function and/or convexities in the borrowing cost function that create hedging opportunities as in Froot et al. (1993). While both these assumptions may play an important role for the demand of insurance, we think that they only partially justify why a country may decide to rely on expensive insurance. Indeed, risk aversion does not necessarily transfer from individuals to countries and the production smoothing argument may not be always a critical one in dealing with fat tail events such as natural disasters.

For the above reasons, in our model we assume that (i) agents are risk neutral, to abstract from consumption smoothing motives, and (ii) that the premium requested to insure the infrastructure stock against a natural disaster is higher than the expected return of rebuilding the same infrastructure, to rule out hedging motives. In such a setting, demand for insurance arises because of its "catalytic" role on external finance. By this we mean that, by guaranteeing resources that limit the economic contraction in the aftermath of

[1] By exposure we denote the probability of being hit by a natural disaster and by vulnerabilty the expected loss associated with any of such disaster.

[2] See Elrich and Becker (1972).

[3] See Hofman and Brukoff (2006) for a survey of the insurance opportunities available to developing countries against natural disasters.

[4] This was the first tranche of a 450 million US dollars insurance coverage plan. Payment are triggered if a earthquake of magnitude 7.5 or 8 hits some predefined zones of the country. See Nell and Richter (2004) for a discussion of the of parametric insurance, that is of insurance policies with payments linked to measurable events such as the magnitude of an earthquake, or the wind-speed of an hurricane.

[5] Note this is still considerably lower than industry averages that range from 5 to 6 times the fair price.

[6] See Ibragimov et al. (2008).

[7] For a comprehensive discussion of the market for catastrophe risk, see Froot (2001). For a discussion of the securitization of catastrophe risk and the development of a catastrophe bond market, see Doherty (1997).

a shock, insurance makes default relatively less likely. This relaxes a country's borrowing constraint and enhances its access to capital markets both at the time of the shock and in normal times. For large rare events such as the ones we consider here, such a benign effect may well outweigh cost considerations.

Of course, not all countries benefit the same from catastrophe insurance. The ones that benefit the most are those low to medium income countries that have limited access to the international capital market and face financial constraints either when they are hit by a shock or in its anticipation. Conversely, catastrophe insurance should not appeal to either very poor countries without access to capital markets that cannot profit from such catalytic effect, or to rich countries that preserve market access even in the aftermath of a large disaster and thus have no reason to pay the expensive insurance premium.

Importantly, some of the middle income countries that benefit from contracting insurance may still qualify for the cheap official emergency lending typically available in the aftermath of a catastrophe. Would this justify the observed underinsurance among low to middle income countries that could potentially benefit from insurance? To answer this question, we augment our model by introducing a multilateral catastrophe lending facility that guarantees access to reconstruction funds at the risk-free interest rate in the event of a natural disaster. To abstract from credit risk, we assume that multilateral lenders enjoy a preferred creditor status that ensures repayment.[8]

The introduction of the facility weakens but does not eliminate the demand for catalytic insurance. The reasons relates to the difference between insurance and lending: while the former entails a positive transfer in bad times, the latter requires repayment of the multilateral loan, tightening the borrowing constraint and crowding out private lending. This, in turns, implies that the stronger the credit constrained faced by the country, the larger the advantage of catastrophe insurance relative to pre-arranged emergency lending, and the larger the demand for the former.[9]

2 The model

Consider an economy endowed with a two-factor Leontief technology:

$$Q = \rho \min\{\min\{1, L\}, K\} \tag{1}$$

to produce a single consumption good. The first factor, denoted by L, can be thought of as infrastructure, which we assume to be at its maximal level $\overline{L} = 1$ at the beginning of the production cycle ($t = 0$). The second factor, K, represents installed productive units (or capital, for short), which we assume to be zero at $t = 0$, and needs to be externally financed (see below). ρ ($\rho > 1$) denotes a total factor productivity parameter.

The timing of the model is as follows:

At time $t = 0$, the country issues bonds for an amount D_0 to finance capital investment $K = D_0$. The gross borrowing cost i is assumed to be equal to the risk free rate r_f (which we normalize to 1 without loss of generality) plus a risk premium δ, itself a function of the probability that the country defaults on the bond (under a zero recovery assumption, see below). We thus have that $i = 1 + \delta$.

In the interim period,[10] $t = 1$, with a probability π–which we assume to be "small enough" throughout the paper–the country is hit by a natural disaster that destroys a fraction $\beta > 0$ of its infrastructure. Faced with such a negative shock, the country has the option to issue new bonds for an amount D_1 to finance infrastructure reconstruction, so that $L = 1 - \beta + D_1$.

At the end of the production cycle, $t = 2$ output is realized and consumption takes place.

[8] As documented by Jeanne and Zettelmeyer (2003), multilateral lending to middle income countries is virtually default risk-free. However, for our purposes it is sufficient to assume that the associated default costs are higher than for private claims.

[9] To our knowledge, these issues have not been yet examined in the economic literature. By contrast, there is growing economic literature assessing the economic costs of natural disasters. See, inter alia, Mauro (2006), Ramcharan (2005), Toya and Skidmore (2006).

[10] For the sake of simplicity, and without great loss of generality, we assume that the interim period is close enough to the initial period so that the borrowing costs are the same in both periods.

Denoting by the subscript b and g "bad" and "good" states of nature, according to whether the shock occurs or not, output X in period 2 can be written as:

$$X_g = \overline{x} + Q_g = \overline{x} + \rho \min\{1, D_0\}, \tag{2}$$

$$X_b = \overline{x} + Q_b = \overline{x} + \rho \min\{1 - \beta + D_1, D_0\}, \tag{3}$$

where $\overline{x}$, $\overline{x} \in \Re^+$, denotes period 2 endowment which we take as a proxy for the country's income level. The country's ability to raise new funds D_1, after suffering the adverse shock, depends on its access to capital market, which depends on its creditworthiness and thus on its perceived willingness to repay its debt obligations. Specifically, following the "old" sovereign debt literature à la Cohen and Sachs (1986), we assume that a default causes the country to lose a share $\gamma < 1$ of its current output X, a loss that is not fully appropriated by the lenders. For simplicity, and without great loss of generality, we then assume that no part of this lost income accrues to the lender.

The country thus faces two distinct borrowing constraints depending on whether default is avoided altogether, or it is expected only in the event of an adverse shock. In the first case, the constraint requires that default costs, in bad states, exceed the cost of servicing the debt, or:

$$D_0 + D_1 \leq \gamma X_b = \gamma \left(\overline{x} + \rho \min\{1 - \beta + D_1, D_0\}\right). \tag{4}$$

Of course, if the country does not default in bad states it would a fortiori not default in good ones. In the second case, instead, (4) does not hold, lenders anticipate default in bad states and charge a risk-adjusted interest rate $i = \frac{1}{1-\pi}$. The borrowing constraint that ensures that default is avoided in good states can then be written as:

$$\frac{D_0}{1-\pi} \leq \gamma \left(\overline{x} + \rho \min\{1, D_0\}\right). \tag{5}$$

Finally, we assume that

$$\frac{1}{\gamma} > \rho > \frac{1}{1-\pi}, \tag{6}$$

where the first inequality implies that investment increases default costs by less than it increases debt so that a country without endowment has no access to finance. The second inequality, instead, ensures that investing in period 0 is always optimal.

In our model, consumers are risk neutral, and policy makers maximize expected income Y,

$$E^j(Y) = (1-\pi)Y_g^j + \pi Y_b^j \tag{7}$$

where superscript $j \in \{d, nd\}$ denotes whether the country defaults if hit by an adverse shock or it does not, and

$$\begin{aligned} Y_g^{nd} &= X_g - D_0; \qquad (8)\\ Y_b^{nd} &= X_b - D_0 - D_1; \\ Y_g^{d} &= X_g - \frac{1}{1-\pi} D_0; \qquad (9)\\ Y_b^{d} &= (1-\gamma) X_b. \end{aligned}$$

Note that in this set-up, income and welfare are mostly determined by the borrowing constraints, and the latter are, in turn, a function of endowment $\overline{x}$. This implies that they are more likely to bind in poor countries than in richer ones. In fact, given that default costs are proportional to total income, which, in turn, depends on endowments, the latter plays the role of "implicit" collateral to the bond issuance: richer countries have more to lose if default is the avenue of choice.

We exploit this dimension in the characterization of the general solution of the "benchmark case" by distinguishing five intervals according to the value of the country's endowment $\overline{x}$. In the main text, we will provide an intuitive characterization of our main results and we refer the reader to the Appendix for the complete analytical treatment.

2.1 Benchmark Scenario

In high-income countries,[11] $\left(\overline{x} \geq x_1^B\right)$ default costs are large enough to ensure that the borrowing constraint (4) is never binding. As a result, the country can borrow at the risk-free rate the optimal amount $D_0 = 1$ in period 0, and the optimal amount $D_1 = \beta$ in period 1, if it is hit by a shock. Production is always maximized, and so is expected income.

In less rich countries $(x_2^B < \overline{x} < x_1^B)$, endowment no longer provides enough "collateral" to ensure that borrowing constraints are always slack. As a result, the country cannot borrow $D_0 = 1$ in period 0 and $D_1 = \beta$ in the event of a shock. In this case, policy makers have a choice between maximizing period 0 investment or "underinvesting" initially, that is choosing a $D_0 < 1$ in order to "save" additional access to finance should an adverse shock occur in period 1. We can show that, if the shock is rare enough–so that the underinvesting "self insurance" option is extremely expensive in terms of lost consumption–the country always chooses to maximize period 0 investment at the expense of period 1 additional access to finance in the event of an (unlikely) adverse shock. We thus have that $D_0 = 1$, $D_1 < \beta$.

If income further decreases $(\overline{x} < x_2^B)$, the borrowing constraint now prevents the country from financing the optimal amount of capital in period 0 and avoiding default in bad states. As before, for rare shocks, the country chooses to maximize borrowing in period 0 at the expense of borrowing after a shock in period 1, so that we have $D_0 \leq 1$, $D_1 = 0$. The relevant borrowing constraint (4) now becomes

$$D_0 \leq D_0^{nd} \equiv \gamma(\overline{x} + \rho(1-\beta)) < 1, \tag{10}$$

where the superscript nd denotes non default in the bad state. Note, however, that the financially constrained country now has the option to increase its indebtedness up to the level that ensures repayment in good (but not in bad) states. Specifically, it can borrow up to (5), which now becomes:

$$D_0 \leq (1-\pi)\,\gamma\,(\overline{x} + \rho D_0) \tag{11}$$

from which

$$D_0 \leq D_0^d \equiv \min\left\{\frac{(1-\pi)}{1-(1-\pi)\gamma\rho}\gamma\overline{x}; 1\right\}. \tag{12}$$

where the superscript d denotes default in the good state. It can be shown that for sufficiently high endowment values $(x_3^B \leq \overline{x} < x_2^B)$ total income is maximized by the lower (default-free) level of indebtedness, whereas for lower values of endowment $(x_3^B > \overline{x} \geq x_4^B)$ the country chooses the higher level of indebtedness, borrowing and investing D_0^d in period 0, and defaulting whenever it is hit by a shock.

Finally, poor countries $(\overline{x} < x_4^B)$ choose, again, to restrain their borrowing in period 0 so as to avoid default if hit by a shock in period 1.

The above analysis is summarized in **Figure 1.a**, where we plot D_0 and D_1 as a function of initial income $\overline{x}$, setting the rest of the intervening parameters at reasonable (albeit arbitrary) values.[12] Intuitively, for rich countries $(\overline{x} \geq x_1^B)$ creditworthiness is never a problem: endowments provide enough implicit collateral to ensure access to finance to exploit investment opportunities in good states, and to fully rebuild the infrastructure in bad states.

By contrast, all other countries face a trade-off between the amount they can invest in period 0 and what they can invest in period 1 if they are hit by a shock. For rare events, countries are better off investing more in period 0, even if this means losing access to finance in the (unlikely) event of a shock in period 1. In this context, relatively rich upper middle-income countries $(\overline{x} \geq x_2^B)$ can still (partially) finance the rebuilding of infrastructure in period 1. This option is lost when $\overline{x} < x_2^B$.

Moreover, because for $\overline{x} < x_2^B$ countries are forced to underinvest in period 0 in order to avoid default in period 1, they face the choice between borrowing more today and defaulting tomorrow in the event of a shock, and borrowing less today and avoiding default costs tomorrow; a decision that ultimately depends on the extent to which investment can be expanded by accepting the higher risk-adjusted interest rate.

[11] The exact values of the thresholds are provided in the Appendix.

[12] In particular, we assume that $\pi = 0.02, \gamma = 0.3, \rho = 1.25$, and $\beta = 0.4$.

In the case of middle-income countries with good access to capital ($x_3^B \leq \overline{x} < x_2^B$), the additional resources do not justify the higher rate, and default is avoided. These resources becomes relatively more valuable as endowments decline and the financing gap widens so that, for lower middle-income countries ($x_3^B > \overline{x} \geq x_4^B$), overborrowing is the preferred strategy. However, because of the higher interest rate charged to the overborrowing country, the financial constraint tightens faster in this case ($\frac{\partial D_0^d}{\partial \overline{x}} > \frac{\partial D_0^{nd}}{\partial \overline{x}}$), which, in turn, explains why low-income countries ($\overline{x} < x_4^B$) prefer to limit investment to avoid default.

3 Insurance

The presence of borrowing constraints–that limit initial investment, the post-shock rebuilding effort, or both–create efficiency losses in all but the richest countries. The natural arrangement to mitigate the problem is an insurance contract that, in exchange of a premium, pays off the country in bad states. To discuss the effect of the availability of such an insurance policy, we assume that in period 0 the country has the option to purchase an insurance contract that pays off an amount Z in the interim period in case the country is hit by a negative shock. To buy one unit of insurance, the country pays a premium $\varphi = \pi\nu$. π is the net present value of a unit expected insurance outlay, and ν is a margin that reflects, inter alia, intermediation costs and a risk premium (alternatively, the insurer's cost of capital, including potential increases if the event materializes). We assume that the insurance premium is paid up front and financed through debt issuance. In addition, we also assume that

$$\nu \in \left[\rho, \rho \frac{1-\gamma}{1-\gamma\rho+\pi(\rho-1)}\right], \tag{13}$$

a non-empty interval for small enough π ($\pi < \gamma$), to explicitly model the fact that insurance is expensive[13] but not so expensive that the country would never buy it. Furthermore, to simplify the analysis, we also assume that

$$\nu < \frac{1}{\beta}$$

which guarantees that no default occurs in the insurance case.[14]

The expected income of a country that in period 0 borrows $D_0 + \varphi Z$ to invest K_0 and to purchase Z units of insurance is

$$E(Y) = \overline{x} + \rho(1-\pi)K_0 + \pi\rho \min\{K_0, (1-\beta) + Z + D_1\} - (K_0 + \varphi Z + \pi D_1). \tag{14}$$

With this new instrument available, we revisit the country's choices as a function of income levels. In the case of rich countries ($\overline{x} \geq x_1^B$) no insurance is needed to attain the optimum (the borrowing constraint is not binding). Moreover, because $\nu > 1$, the effective cost of insurance exceeds that of international capital, and no insurance is purchased. The solution is then identical to benchmark case.

For lower values of $\overline{x}$ ($x_1^B > \overline{x} \geq x_2^I$) the borrowing constraint limits period 1 borrowing. As in the benchmark case, for rare events the country always chooses to maximize period 0 investment (which in this case attains the optimal $D_0 = 1$), so that period 1 is the residual variable. In this regard, insurance plays a complementary role: by ensuring the availability of resources in the aftermath of a shock, it increases output in bad states and, through this channel, relaxes the borrowing constraint (4) that becomes:

$$(D_0 + \pi\nu Z) + D_1 \leq \gamma(\overline{x} + \rho((1-\beta) + Z + D_1)) \tag{15}$$

so that

$$D_1 \leq D_1^{nd}(Z, D_0) \equiv \frac{\gamma(\overline{x} + \rho(1-\beta)) - D_0 + (\gamma\rho - \pi\nu)Z}{(1-\gamma\rho)}, \tag{16}$$

[13] This stylyied contract applies more directly to the case of a parameterized CAT bond with principal Z and coupon φ, which, in the case of a verifiable natural disaster, virtually eliminates the need for costly state verification. Standard CAT insurance, by contrast, are typically based on actual losses and cover pre-specified layers, defined by a deductible or "retention" (below which no loss is covered) and a "limit" above which no loss is covered.

[14] Such assumption does not alter the qualitative results of our analysis (insurance always reduces the likelihood of default) but limit the number of cases that we have to analyze.

and

$$\frac{\partial D_1^{nd}}{\partial Z} = \frac{\gamma\rho - \pi\nu}{1-\gamma\rho} \geq 0. \tag{17}$$

This implies that insurance has a "catalytic" effect on private lending: by purchasing insurance the country enhances access to the international capital market in bad states. This effect explains why the country might be willing to purchase insurance even if it is expensive relative to capital markets. More precisely, the derivative of expected income on insurance is given by:

$$\frac{\partial E(Y)}{\partial Z} = \pi\left(-(\nu-\rho) + (\rho-1)\frac{\gamma\rho-\pi\nu}{1-\gamma\rho}\right) > 0 \tag{18}$$

where the expensive premium (the first RHS term) is counterbalanced by the positive catalytic effect (the second RHS term). On the other hand, given its costly nature, the country would purchase insurance only as a complement to market funds, so that

$$Z \equiv \beta - D_1^{nd}, \tag{19}$$

since increasing coverage beyond Z would simply substitute expensive insurance for less costly debt. In this way, insurance fills in for private markets, both crowding in additional private funds and providing the resources that the market does not offer to allow the country to insure against the shock.

For lower income countries ($\overline{x} < x_2^I$), insurance will no longer have a catalytic effect, as the country cannot borrow in period 1. However, the country may still be interested in buying insurance. Why? To relax its borrowing constraint in period 0, which now becomes

$$(D_0 + \pi\nu Z) \leq \gamma(\overline{x} + \rho\min\{D_0, (1-\beta)+Z\}), \tag{20}$$

from which we have that:

$$D_0 \leq D_0^{nd} \equiv \gamma\left(\overline{x} + \rho(1-\beta)\right) + (\gamma\rho - \pi\nu)Z. \tag{21}$$

It is easy to check that for small values of π, $\frac{\partial D_0^{nd}}{\partial Z} > 0$ and $\frac{\partial E(Y)}{\partial Z} > 0$. In other words, since insurance increases access to capital market in period 0, the country fully insures the productivity of period 0 investment, that is, purchases insurance for an amount $Z = D_0^{nd} - (1-\beta)$ so as to bring infrastructure to $L = K = D_0$ in the event of a shock.

It is important to note that it is never in the best interest of the country to overborrow at the risk of defaulting in bad states and purchasing insurance. The intuition is the following. Consider a country that overborrows and defaults in bad states; the anticipation of default eliminates the catalytic effect of insurance (through its role in the now irrelevant (4)). In turn, without this effect, insurance is simply too expensive and no insurance would be purchased if default in bad states is inevitable. Note that insurance does not work as a substitute for lending. Rather, it pays off to insure only if the country reaps the crowding-in benefits, for which insurance funds need to work as collateral to prevent default in bad states. If default in bad states is anticipated, the collateral value of insurance vanishes.

Finally for poor countries ($x_3^I > \overline{x}$), access to insurance no longer plays a role as the catalytic effect disappears because the lack of creditworthiness limits investment opportunity in period 0.

The previous analysis is summarized in **Figure 1.b**, where we plot debt and insurance outlays under the insurance case for the same parameter as in **Figure 1.a**. In addition, we assume that the overhead parameter $\nu = 1.3$.

As we already pointed out, for high income countries the borrowing constraint does not bind, no insurance is purchased, and the results are the same as in the benchmark case. Less rich countries ($x_1^B > \overline{x} \geq x_2^I$), instead, start purchasing insurance to increase their ability to rebuild infrastructure in the aftermath of the shock. Notice that in this case insurance plays two roles. On the one hand, by ensuring the availability of "prefinanced" reconstruction funds, it increases output in bad states and, in turn, default costs, crowding in private lenders. As a result, in this region, insurance enlarges the amount of resources available in the aftermath of the shock relative to the benchmark, and complements private funds. Also, the more credit constrained a country is, the more insurance it buys. Relatively poorer countries ($\overline{x} < x_2^I$), instead, are less affected by the shock (because of their limited capital investment in period 0)), and this negatively affects their demand for insurance.

4 A catastrophe lending facility

The previous analysis focused on the "supply side" of the problem showing that the availability of insurances allows a country to relax the borrowing constraint–albeit at or despite a considerable cost. However, because large events of a systemic nature such as natural disasters involve massive economic losses and affect a large number of people, they typically trigger ex-post government intervention which agents correctly anticipate. This often creates Samaritan dilemma type of problems that lead individuals to underinvest in catastrophe insurance. Similarly, at the international level, catastrophes in low-income countries elicit an almost immediate reaction by the international community in the form of (often concessional) loans for social expenditure and reconstruction. Why would a country bear the exorbitant insurance premiums if it is likely to have access to official resources at a small cost? Is this version of the Samaritan's dilemma what is behind the scarcity of catastrophe insurance in middle- and low–income countries?

We can easily adapt our model to look into this issue and examine whether insurance is still purchased by the country in presence of a catastrophe lending facility offering unlimited funds at the risk-free rate in the event a shock. For expositional purposes, it is easier to tackle this question in two steps, solving first for the lending facility in the absence of insurance, and then introducing the insurance.

Consider now the case in which a *multilateral lender* offers a catastrophe lending facility, as in the case of the recent approved World Bank CAT DDO facility,[15] from which a country can draw (only) in the event of a shock. It is easy to show that this facility cannot be offered by private markets because loan amounts will be restricted by the borrowing constraint in exactly the same way D_1 was in the previous case. However, a multilateral lender could in principle exploit its preferred creditor status to provide access in period 1 beyond what the borrowing constraint allows. Indeed, preferred official creditors (the government at the national level, multilaterals and donors at the international level) are the ones that usually come to the rescue after large natural disasters.

In order to represent the preferred creditor status of the multilateral lender, we assume that defaulting on the multilateral is prohibitively costly so that multilateral loans are always repaid. Therefore, in this case, selective default on private creditors could be an equilibrium outcome.[16]

Given the debt D_0 (with private lenders) and M (with official lenders), in period 2 the country faces two choices: repay or default on bonds.

More formally, in period 2, the sovereign does not default on bonds if, and only if

$$D_0 \leq D_0^{nd}(M) = \gamma\left[\overline{x} + \rho\left(1 - \beta + D_1 + M\right)\right] - D_1 - \gamma M, \tag{22}$$

whereas the unconstrained M is set to maximize period 2 output, i.e., $M = D_0 - (1 - \beta)$.

Replacing M into (22), we obtain

$$D_0 \leq D_0^{nd} = \frac{\gamma\left(\overline{x} + 1 - \beta\right)}{1 - \gamma\left(\rho - 1\right)}. \tag{23}$$

Note that the fact that M is chosen ex post (i.e., the country cannot commit not to borrow from the facility in period 1) simplifies the problem, which now boils down to the choice of period 0 borrowing, D_0. Also note that, under the assumption that multilateral and private lending command the same interest rate,

[15] The CAT DDO is a new financial product offered to middle-income country governments by the International Bank for Reconstruction and Development (IBRD), part of the World Bank Group.

[16] We implicitly assume that the multilateral has no way of conditioning its lending on the continued service of the debt with private lenders, which in our setup simply reflects a sequencing issue: the fact that the contingent loan is disbursed in period 1, before the bond matures. However, we come back to this point in the final section.

the actual composition of period 1 lending is immaterial for the current analysis. Then, without loss of generality, we can set $D_1 = 0$.

In the non default case, expected income can be expressed as

$$E(Y) = \overline{x} + (\rho - 1) D_0^{nd} - \pi \left[D_0^{nd} - (1 - \beta) \right]. \tag{24}$$

However, the country can also borrow beyond the limit imposed by (23) and, after a shock, withdraw from the facility and default on the bond. In this case, expected income is given by

$$E(Y) = (1 - \pi\gamma) \left[\overline{x} + (\rho - 1) D_0^d \right] + \pi (1 - \gamma)(1 - \beta) \tag{25}$$

As before, the equilibrium can be characterized by income levels. In the case of rich countries ($\overline{x} > x_1^B$) the borrowing constraint is not binding: the country borrows and invests $D_0^{nd} = 1$ in period 0, and $D_1 = \beta$, in period 1 in the event of a shock. However now, less rich countries $\overline{x} \in [x_1^M, x_1^B]$, can borrow from the facility as much as they need because defaulting on it is not an option.

In the case of poorer countries, ($\overline{x} \in [x_2^M, x_1^M]$), period 0 borrowing constraint binds: $D_0^{nd} < 1$ and, as a result, $M = D_0^{nd} - (1 - \beta) < \beta$. For lower level of endowments ($\overline{x} \in [x_3^I, x_3^M]$) the credit constrained country chooses to borrow $D_0^d = \min \left\{ \frac{(1-\pi)\gamma\overline{x}}{1-(1-\pi)\gamma\rho}, 1 \right\}$ in period 0 at a risk-adjusted rate $i = \frac{1}{1-\pi}$, and, if hit by the shock, borrows $M^d = D_0^d - (1 - \beta)$ from the contingent credit line in period 1, and defaults in period 2 on its obligation to private creditors. The intuition is similar to that in the benchmark case, except that now the overborrowing country still has access to financial resources in period 1. Indeed, overborrowing also increases output in bad states, since reconstruction funds are not restricted by the borrowing constraint and increase linearly with period 0 investment. For this reason, default has a smaller impact on income than in the benchmark. Finally, for the same reason as before, low income countries ($\overline{x} \in [x_3^I, x_3^M]$) choose to avoid default and borrow from the facility as long as they face capital needs in period 1

A visual comparison of how the multilateral facility (see **Figure 1.c**) compares with the benchmark case reveals that the presence of the contingent credit line narrows the interval in which the country chooses to default. This is so because the contingent credit line increases the value at stake in case of a default. Given that default costs in this setup are proportional to output, the benign output effect of the contingent line increases the cost of defaults and reduces their incidence–even though the defaulting country still has access to the multilateral loan.

In turn, comparing with the insurance case, borrowing in period 0 is never higher under the lending facility. Again, the intuition is relatively straightforward: whereas the insurance premium entails a transfer from good to bad states (and, in particular, is arbitrarily small for rare events), the catastrophe loan transfers the cost of the shock intertemporally within bad states (that is, states marked by the occurrence of the shock), creating a sharp asymmetry between good and bad states, and tightening the borrowing constraint associated with the latter. Hence, the lower borrowing amounts (due to the crowding out of period 0 bond borrowing by period 1 multilateral lending) and the positive probability of default.

Regarding this point, note that for simplicity we assumed that the lending facility extended one-period loans. While this realistically reflects the short-run nature of most emergency and concessional lending, it bears the question of whether a longer loan can substitute insurance in those cases in which, because of market imperfections or political economy reasons, supply or demand for insurance is likely to be insufficient. More specifically, can a 1 in 30 years event be covered indistinctly by insurance and by a 30-year contingent loan?

According to the previous analysis, it cannot. A country that optimally borrows from the facility after it is hit by a shock inherits the full stock of debt, irrespective of the duration of the loan. In other words, since default in this case is not the result of a liquidity crisis but rather the consequence of a cost-benefit analysis, it is the stock of debt rather than its flow cost that determines the decision.

Consider now the case in which the country has access to both insurance and the lending facility. Would the country still purchase insurance in this case, or would it rely entirely on catastrophe lending? In other words, does the facility make the supply of insurance redundant for the country?

To answer the question, first note that for relatively rich countries ($\overline{x} \in [x_1^M, x_1^B]$), borrowing from the facility is clearly superior to insurance, because it allows the country to circumvent the borrowing constraint at a lower cost. On the other hand, it is easy to verify that, for $x < \overline{x}_1^M$, insurance is always demanded.

In particular, for $x \in [\overline{x}_2^I, \overline{x}_1^M]$, see **Figure 3a**, the country's problem consists in investing $L_0 = 1$, and $L_1 = \beta$, at the lowest cost, which in turn implies minimizing the amount of (costly) insurance compatible with that objective. The borrowing constraint (4) now becomes:

$$(D_0 + \pi\nu Z) \leq \gamma(\overline{x} + \rho\left((1-\beta) + Z + M\right)) - \gamma M, \tag{26}$$

and substituting

$$M = D_0 - (1-\beta) - Z \tag{27}$$

(26) can be rewritten as:

$$D_0 \leq \frac{(\overline{x} + 1 - \beta + Z)\gamma - \pi\nu Z}{1 - \gamma(\rho - 1)}. \tag{28}$$

It is easy to verify that within this interval, both insurance and multilateral lending coexist. In addition, for $\overline{x} \leq \overline{x}_2^I$, low period 0 investment levels make the lending facility redundant, and insurance becomes the only source of funding in bad states. Thus, we are back to the insurance case discussed previously.

In summary, the demand for insurance is crowded out by the presence of the facility only for those relatively high income countries for which the facility is enough to lift the borrowing constraint. However, because multilateral lending crowds out access to capital markets in period 0, insurance still plays a helpful role reducing the burden of period 1 debt, thereby relaxing the borrowing constraint. In other words, while the Samaritan's dilemma considerations eliminate the need for insurance as a source of funds in bad states, it does not eliminate its catalytic role in good states.

5 Welfare analysis

So far, we have concentrated on the implications of the models in terms of access to finance in both states. Naturally, there is more to this exercise than simply comparing access. Indeed, an evaluation of the different alternatives would have to ponder their consequences in terms of expected income which, without great loss of generality, we perform graphically for the set of parameters used in the figures. Our welfare analysis is summarized in **Figure 2**. The top panel (2a) of the figure plots net income from production in good and bad states (that is, output net of borrowing costs and endowments, or $Y_{g,b} - \overline{x}$), for each of the three main scenarios under study: the benchmark, up-front insurance and ex-post catastrophe lending. The second panel (2b) does the same for expected income. Not surprisingly, both insurance and catastrophe lending are (weakly) superior to the benchmark: income under each alternative (and in both states) is always greater or equal than in their absence. But their relative benefits differ according to the country's endowment.

If access to finance is not critical (richer countries), the insurance option yields a lower expected income than the less expensive multilateral lending facility. However, in the case of low-middle income countries the multilateral facility may, at the same time, crowd out private lending and be ineffective in avoiding costly default. Since for these countries access to finance is critical it is not surprising that higher levels of expected income are associated with the insurance option. What is somehow more surprising is that for a large set of endowment values a country may enjoy higher income in both states of nature if it relies on insurance rather than on the multilateral facility.

Such welfare trade-offs are clearly illustrated in **Figure 3**, where we cast a closer look at the situation in which insurance and the catastrophe lending coexist. As can be seen, the demand for insurance kicks in at the endowment level for which the borrowing constraint starts limiting investment in period 0. Thus, by crowding in private lending in period 0, insurance enables a financially constrained country to reach the optimal level of investment, albeit at a premium that detracts from the optimal expected income.

A potentially undesirable characteristic of the lending facility examined above is that it involves a multilateral institution lending to a country at a time when the country is expected to default on its private creditors. Unlike implicit arrangements, an explicit facility could still condition access to the facility ex ante, so as to make sure that the borrower has the incentives to avoid default.[17] This is not far from standard multilateral practice: multilateral loans are often granted provided that the recipient country meets certain debt sustainability criteria.[18] In this way, the official lender ensures that the country does not take the new money the minute before it defaults on third parties. Intuitively, to the extent that overborrowing excludes the country from the facility, this new condition should detract from the incentives to default, and reduce its incidence.

The solution for a *contingent* catastrophe lending facility (contingent on not defaulting on the private sector) does not differ much from the one presented in the previous section (see Appendix). Interestingly, a comparison between the contingent and the uncontingent facility reveals the latter to be better, at least in terms of expected income, see **Figure 4**. The reason is that, for those endowment levels for which the two differ, the contingent facility saves the default costs at the expense of leaving the country underfinanced after an adverse shock. However, because the shock is exogenous, the situation involves no moral hazard and no value is created by reducing the incidence of default. On the contrary, the punishment (exclusion from the facility) translates in a lower overall welfare.

6 Conclusion

In this paper, we have shown that catastrophe insurance, even a very expensive one, can be an effective instrument to relax a low to middle income country's borrowing constraint and alleviate the economic costs in the event of a natural disaster.

We have also shown that the presence of emergency lending, either as pre-arranged credit lines or in the form of concessional reconstruction funds typically made available by multilateral institutions and donors, does not completely eliminate the benign catalytic effect of insurance. This is because the insurance and the credit line differ in one crucial aspect: the loan is has to be repaid after a bad shock, while the cost of insurance is transferred to good times through the payment of a premium. As a result, income (and consumption) volatility is bound to be lower with insurance than with concessional lending. Thus, if income smoothing (from which we deliberately abstracted in this paper) were to play a role in the policy choice function, insurance would become even more appealing –which would only add to the case for insurance that follows from our model.

On this last point, a qualification is in order: while we assumed that the ex-post credit line is not concessional and that the lender is paid the risk-free cost of capital, disaster aid is often highly concessional in reality. Our main results would still hold true for a limited level of concessionality. Trivially, should the country expect to receive pure grants in the event of a catastrophe (in other words, an implicit premium-less insurance), demand for CAT insurance would disappear.

Finally, while insurance (particularly when reasonably priced) would seem the logical option for disaster-prone low to middle income countries, the fact that it entails a payment up front in exchange for an infrequent positive transfer makes this type of arrangement a political hard sell. Such political economy considerations cannot be ignored from a positive perspective, and may ultimately explain why we see overinsurance in practice. Thus, from a policy perspective, a multilateral catastrophe lending facility may be the only feasible alternative even for those cases for which CAT insurance is the optimal choice.

[17] To enhance incentives without distorting its automatic nature, the facility could involve temporary subscriptions on a rolling basis, to ensure that the country is not cut off overnight but still faces frequent exams.

[18] However, in the aftermath of a natural catastrophe it is not unlikely that other creditors (especially bilateral) agree to write-off part of their credit to allow multilateral lending.

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7 APPENDIX

7.1 The benchmark case

7.1.1 Case 1B: $\overline{x} \geq x_1^B \equiv \frac{1+\beta-\gamma\rho}{\gamma}$

Setting $D_0 = 1$, and $D_1 = \beta$, it is immediate to verify that (4) becomes

$$1 + \beta \leq \gamma(\overline{x} + \rho),$$

and is satisfied if $\overline{x} \geq x_1^B$.

In turn, $K = D_0 = 1$, $L = D_1 = \beta$ yield

$$E_{1B}^{nd}(Y) = \overline{x} + \rho - (1 + \pi\beta).$$

7.1.2 Case 2B: $x_1^B > \overline{x} \geq x_2^B \equiv \frac{1-\gamma\rho(1-\beta)}{\gamma}$

If $\overline{x} < x_1^B$, the country cannot borrow $D_0 = 1$ in period 0 and still have access to $D_1 = \beta$ in bad states. Therefore, it faces a trade-off between maximizing period 0 investment, or "underinvesting" initially in order to "save" additional access in bad times.

Note that in equilibrium $L \leq K \Longrightarrow 1 - \beta + D_1 \leq D_0$ so that the relevant borrowing constraint (4) can be written as:

$$D_0 + D_1 \leq \gamma\left(\overline{x} + \rho((1-\beta) + D_1)\right),$$

so that:

$$D_1 \leq D_1^{nd} \equiv \frac{\gamma\overline{x} + \gamma\rho(1-\beta) - D_0}{1 - \gamma\rho}.$$

In turn, we can express the trade-off between increasing consumption in good and bad states in terms of the country's problem at time 0:

$$\begin{aligned} \max_{D_0} E^{nd}[Y(D_0, D_1^{nd})] &= \overline{x} + \rho\left((1-\pi)D_0 + \pi(1-\beta)\right) - D_0 + \pi(\rho - 1)D_1^{nd} \\ \text{subject to } 0 &\leq D_1^{nd} \leq D_0 - (1-\beta) \end{aligned}$$

It is then easy to verify that, for π low enough,

$$\frac{\partial E(Y)}{\partial D_0} = \rho(1-\pi) - 1 - \pi(\rho - 1)\frac{1}{1-\gamma\rho} > 0, \tag{29}$$

which indicates that the country maximizes period 1 investment ($D_0 = 1$) at the expense of lower investment in the event of an adverse shock.

Finally, we need to verify that the country can invest a positive amount in the second period while avoiding default. This condition can be written as:

$$D_1^{nd}\Big|_{D_0=1} = \frac{\gamma\overline{x} - (1 - \gamma\rho(1-\beta))}{1-\gamma\rho} \geq 0 \Longleftrightarrow \overline{x} \geq x_2^B \equiv \frac{1-\gamma\rho(1-\beta)}{\gamma}. \tag{30}$$

In sum, for $\overline{x} \in [\overline{x}_1, \overline{x}_2]$, we have that $K^{2B} = D_0 = 1$, and, $L^{2B} = D_1 = \frac{\gamma\overline{x} - (1-\gamma\rho(1-\beta))}{1-\gamma\rho} < \beta$, and

$$E_{2B}^{nd}(Y) = \overline{x} + \rho(1 - \beta\pi) + \pi D_1(\rho - 1) - 1.$$

7.1.3 Case 3B: $x_2^B > \overline{x} \geq x_3^B \equiv \frac{\rho-\rho\gamma-1}{\gamma(\rho-1)(1-\pi)} - (1-\beta)\rho$

It follows from (30) that, in this interval, $D_1 = 0$. Moreover, the country cannot borrow the optimal amount of capital in period 0 without risking default if hit by a shock.

From (10) and (12),

$$D_0^{nd} = \gamma(\overline{x} + \rho(1-\beta)) < 1. \tag{31}$$

and

$$D_0^d \equiv \min\left\{\frac{(1-\pi)}{1-(1-\pi)\gamma\rho}\gamma\overline{x}; 1\right\},$$

In turn,

$$D_0^d = 1 \Longleftrightarrow \overline{x} > \widetilde{x} \equiv \frac{1-\gamma\rho(1-\pi)}{\gamma(1-\pi)} \tag{32}$$

and

$$\widetilde{x} \leq \overline{x}_2 \Longleftrightarrow \pi \leq \frac{\gamma\rho\beta}{1+\gamma\rho\beta}, \tag{33}$$

so that for a sufficiently small π, there exists a non-empty interval $[\widetilde{x}, \overline{x}_2]$ such that $D_0^d = 1$. We also have that

$$D_0^{nd} > (1-\beta) \Longleftrightarrow x > \frac{(1-\beta)(1-\gamma\rho)}{\gamma} \equiv \widetilde{x}_a$$

and

$$\widetilde{x}_a < \widetilde{x} \Longleftrightarrow \pi < 1 - \frac{1}{1-\beta(1-\gamma\rho)}$$

so that in the interval $[\widetilde{x}, \overline{x}_2]$, $\min\{D_0^d, D_0^{nd}\} > 1-\beta$ so that the countries have an interest in borrowing if they are hit by the shock.

Finally, we have that:

$$\begin{aligned} E^{nd}Y((D_0^{nd})) &= (1-\pi)(\overline{x} + \rho D_0^{nd}) + \pi\left[\overline{x} + \rho(1-\beta)\right] - D_0^{nd}, & (34)\\ E^d(Y(D_0^d)) &= (1-\pi)(\overline{x} + \rho D_0^d) + \pi(1-\gamma)\left[\overline{x} + \rho(1-\beta)\right] - D_0^d; & (35)\end{aligned}$$

substituting the values for D_0^d and D_0^{nd} from expressions (31) and (32) in these expression we have that:

$$\begin{aligned} \Delta\left(D_0^d, D_0^{nd}\right) &\equiv E^d\left(Y\left(D_0^d\right)\right) - E^{nd}\left(Y(D_0^{nd})\right) \\ &= 1 + \gamma(1-\pi)(\rho-1)(\bar{x}-\rho), \end{aligned}$$

which is linear in $\overline{x}$.

Trivially, for $x = x_2^B$, we have that $D_0^d = D_0^{nd} = 1$, and $\Delta\left(D_0^{nd}, D_0^d\right) = \pi > 0$. On the other hand we have that

$$\begin{aligned} \lim_{x\to\widetilde{x}} \Delta\left(D_0^{nd}, D_0^d\right) &= \lim_{x\to\widetilde{x}} \Delta\left(1, D_0^{nd}\right) \\ &= \rho(\pi - (1-\pi)\beta\gamma(\rho-1)), \end{aligned}$$

expression that is negative if π is small enough. From this, it follows that within the interval there is a unique value of $\overline{x}_3 \in [\widetilde{x}, x_2^B]$ such that $E(Y(D_0^{nd})) - E(Y(D_0^d)) > 0 \Leftrightarrow \overline{x} > x_3^B$. It is then easy to verify that $x_3^B = \frac{1}{\gamma}\left(\frac{\rho-\rho\gamma-1}{(\rho-1)(1-\pi)}\right) - (1-\beta)\rho$, so that in the interval $[\overline{x}_3, \overline{x}_2]$ the country chooses to borrow $K^{3B} = D_0^{nd} = \gamma\left(\overline{x} + (1-\beta)\rho\right)$ and default is avoided. Income is then given by

$$E_{3B}^{nd}(Y) = (1-\pi)\left(\overline{x} + \rho D_0^{nd}\right) + \pi(\overline{x} + \rho(1-\beta)\} - D_0^{nd}. \tag{36}$$

7.1.4 Case 4B: $x_3^B > \overline{x} \geq x_4^B \equiv \frac{(\rho-1)(1-(1-\pi)\gamma\rho)}{(1-\pi)(\rho-1)\gamma-\pi}(1-\beta)$

From the previous proof, it follows that for $\widetilde{x} \leq \overline{x} \leq x_3^B$, $E(Y(D_0^d)) - E(Y(D_0^{nd})) > 0$, and $K_0 = D_0^d = 1$.

Consider now the interval $[\widetilde{x}, \overline{x}_4]$ for which:

$$D_0^d = \frac{(1-\pi)}{1-(1-\pi)\gamma\rho}\gamma\overline{x} < 1,$$

First of all let's verify that in this interval $\min\{D_0^d, D_0^{nd}\} > 1-\beta$. Indeed, we have:

$$\begin{aligned} x_a^d &\equiv \{x : D_0^d = (1-\beta)\} = (1-\beta)\left(\frac{1}{(1-\pi)\gamma} - \rho\right); \text{ and } D_0^d \geq (1-\beta) \text{ if } x > x_a^d; \\ x_a^{nd} &\equiv \{x : D_0^{nd} = (1-\beta)\} = \frac{(1-\beta)(1-\gamma\rho)}{\gamma}; \text{ and } D_0^{nd} \geq (1-\beta) \text{ if } x > x_a^{nd}; \end{aligned}$$

from which we have that

$$x_a^d - x_a^{nd} = \frac{\pi(1-\beta)}{(1-\pi)\gamma} > 0$$

and, in turn, that in the interval $[x_4^B, x_3^B]$, $x_4^B > x_a^{nd} \Longrightarrow \min\{D_0^d, D_0^{nd}\} > 1-\beta$. Indeed,

$$x_4^B - x_a^{nd} = \frac{\pi(1-\beta)(1-(1-\pi)\gamma\rho}{(1-\pi)\gamma((1-\pi)\gamma\rho - (1-\pi)\gamma - \gamma)} > 0 \Longleftrightarrow \pi < 1 - \frac{1}{\gamma\rho}$$

which is always verified for π small enough. We can now substitute the values value into D_0^d, and D_0^{nd} in (34) and (35) to get

$$\begin{aligned} \Delta\left(D_0^d, D_0^{nd}\right) &= \frac{(1-\pi)\gamma\rho\left(1-\beta-\gamma\left(\overline{x}+\rho(1-\beta)\right)(\rho-1)\right) + \pi\left(((1-\beta)\gamma\rho - 1)\rho + \overline{x}\left(1-\gamma(1-\rho)\right)\right)}{1-(1-\pi)\gamma\rho} \geq 0 \\ &\Longleftrightarrow \quad \overline{x} \geq x_4^B \equiv \frac{(\rho-1)(1-\beta)(1-(1-\pi)\gamma\rho)}{(1-\pi)(\rho-1)\gamma-\pi}. \end{aligned}$$

In sum, for $\overline{x} \in [x_4^B, x_3^B]$, $\Delta\left(D_0^d, D_0^{nd}\right) > 0$, and the country chooses to borrow $K_0^{4B} = D_0^d = \min\{\frac{(1-\pi)\gamma\overline{x}}{1-(1-\pi)\gamma\rho}; 1\}$ in period 0, and defaults if hit by a shock in period 1. In this case, income is given by

$$E^{4B}(Y) = (1-\pi)(\overline{x} + \rho D_0^d) + \pi\left(1-\gamma\right)\left(\overline{x} + \rho(1-\beta)\right) - D_0^d.$$

7.1.5 Case 5B: $x_4^B > \overline{x} \geq 0$

Finally, it is easy to check that for $x < x_4^B$, $K_0^{5B} = D_0^{nd} = \gamma(\overline{x} + \rho(1-\beta))$ and the country does not default.

In this case, expected income is given by, in case 3B, by

$$E^{5B}(Y) = (1-\pi)\left(\overline{x} + \rho D_0^{nd}\right) + \pi(\overline{x} + \rho\min\{(1-\beta), D_0^{nd}\} - D_0^{nd}.$$

7.2 Insurance

7.2.1 Case 1I $\left(\overline{x} > x_1^B\right)$

In the case of rich countries ($\overline{x} \geq x_1^B$)no Insurance is needed to attain the optimum (the borrowing constraint is not binding). Moreover, because $\nu > 1$, the effective cost of insurance exceeds that of international capital, and no insurance is purchased.

7.2.2 Case 2I $\left(x_1^B > \overline{x} \geq x_2^{IN} \equiv \frac{1-\gamma\rho+\pi\beta\nu}{\gamma}\right)$

As in case $2B$, the borrowing constraint determines period 1 borrowing. However, unlike in the benchmark, insurance plays a complementary role by increasing the collateral and relaxing the constraint. Specifically, we can write the country's problem at time 0 in terms of D_0, D_1 and Z as:

$$\max_{D_0,Z,D_1} E^{nd}\left[Y(D_0,Z,D_1)\right] = \overline{x} + \left((1-\pi)\rho - 1\right)D_0 + \pi\rho(1-\beta) + \pi(\rho-1)D_1 - \pi\left(\nu-\rho\right)Z \tag{37}$$

subject to the borrowing constraint

$$(D_0 + \pi\nu Z) + D_1 \leq \gamma\left[\overline{x} + \rho\left(1-\beta\right) + Z + D_1\right)] \tag{38}$$

from which

$$D_1 \leq D_1^{nd}\left(Z, D_0\right) \equiv \frac{\gamma\left(\overline{x} + \rho\left(1-\beta\right)\right) - D_0 + \left(\gamma\rho - \pi\nu\right)Z}{\left(1-\gamma\rho\right)}, \tag{39}$$

with $\frac{\partial D_1^{nd}}{\partial Z} = \frac{\gamma\rho-\pi\nu}{1-\gamma\rho} \geq 0$.

Substituting (39) into (37), we have that, for any given Z, and for small π,

$$\pi < \frac{\left(\rho-1\right)\left(1-\gamma\rho\right)}{\rho\left(1-\gamma\rho\right) + \left(\rho-1\right)}, \tag{40}$$

$$\frac{\partial E(Y)}{\partial D_0} = (1-\pi)\rho - 1 - \frac{\pi(\rho-1)}{(1-\gamma\rho)} > 0.$$

which implies that the country maximizes period 0 investment and tells us, in particular, that $D_1^{nd} > 0 \Longrightarrow D_0 = 1$.

Consider the case in which endowment $\overline{x}$ is large enough to allow the country to borrow $D_0 = 1$. Differentiating (37) with respect to Z we obtain:

$$\frac{\partial E(Y)}{\partial Z} = \pi\left(-\left(\nu-\rho\right) + \left(\rho-1\right)\frac{\gamma\rho-\pi\nu}{1-\gamma\rho}\right) > 0$$

where the positive sign comes from (13), so that the country purchases insurance subject to

$$Z \leq Z^{nd} \equiv \beta - \ D_1^{nd}. \tag{41}$$

Finally, substituting (41) into (39) it is easy to verify that:

$$\begin{aligned} D_1^{nd}\left(Z^{nd}, 1\right) \quad &= \quad \frac{\gamma(\overline{x}+\rho) - (1+\pi\nu\beta)}{1-\pi\nu} \geq 0 \\ \iff \quad &\overline{x} \geq \frac{1-\gamma\rho+\pi\nu\beta}{\gamma} \equiv x_2^{IN} \end{aligned}$$

and that

$$Z^{nd} = \frac{1+\beta-\gamma\left(\overline{x}+\rho\right)}{\left(1-\pi\nu\right)} \geq 0 \Longleftrightarrow \overline{x} \leq \frac{1+\beta-\gamma\rho}{\gamma} = x_1^B$$

so that in the interval $\left[\overline{x}_2^{IN}; x_1^B\right]$, $D_0 = 1$, $D_1^{nd} > 0$ and $Z^{nd} > 0$.

7.2.3 Case 3I $\left(x_2^{IN} > \overline{x} \geq x_3^{IN} \equiv \frac{(1-\beta)(1-\gamma\rho)}{\gamma}\right)$

If $\overline{x} < \overline{x}_2^{IN}$, the country fully exhausts its access to capital in period 0, so the "catalytic" effect of insurance is reflected directly in the borrowing constraint determining access in period 0. Specifically, insurance increases income in period 1 (the value at stake in bad states), at a cost proportional to the likelihood of the insured event, relaxing the borrowing constraint that now becomes

$$(D_0 + \pi\nu Z) \leq \gamma(\overline{x} + \rho \min\{D_0, (1-\beta) + Z\}),$$

or

$$D_0 \leq D_0^{nd} \equiv \gamma\left[\overline{x} + \rho\left(1-\beta\right)\right] + \left(\gamma\rho - \pi\nu\right) Z. \tag{42}$$

Substituting D_0^{nd} in the objective function

$$E\left(Y\left(D_0^{nd}, Z\right)\right) = \overline{x} + \left(\left(1-\pi\right)\rho - 1\right) D_0^{nd} + \pi\rho(1-\beta) - \pi\left(\nu - \rho\right) Z$$

we have:

$$\frac{\partial E(Y)}{\partial Z} \geq 0 \Longleftrightarrow \gamma\rho - \pi\nu > \frac{\gamma - \pi}{1-\pi}. \tag{43}$$

which is trivially verified for a small enough π.

This in turn implies that the country always purchase insurance subject to

$$Z \leq Z^{nd} = D_0 - (1-\beta),$$

where Z^{nd} denotes the point at which insurance funds allow the country to rebuild infrastructure to the level $L = K$.

Finally, substituting Z^{nd} in (42) we have that

$$Z^{nd} \geq 0 \Longleftrightarrow D_0^{nd} = \frac{\overline{x}\gamma + (1-\beta)\pi\nu}{1 - \gamma\rho + \pi\nu} \geq (1-\beta) \Longleftrightarrow \overline{x} \geq x_3^{IN} \equiv \frac{(1-\beta)(1-\gamma\rho)}{\gamma}$$

In this interval the country has the option to borrow D_0^d, such that $1 \geq D_0^d > D_0^{nd}$, at a risk-adjusted rate, and default in bad states. If so, the borrowing constraint (5) becomes:

$$\frac{1}{1-\pi}(D_0 + \pi\nu Z) \leq \gamma(\overline{x} + \rho D_0) \tag{44}$$

from which

$$D_0 \leq D_0^d \equiv \min\left\{\frac{\left(1-\pi\right)\gamma\overline{x} - \pi\nu Z}{1 - \left(1-\pi\right)\gamma\rho}; 1\right\}, \tag{45}$$

and expected income modifies to

$$E^d\left(Y(D_0^d, Z)\right) = (1-\gamma\pi)\overline{x} + (1-\pi)\left[\rho - \frac{1}{(1-\pi)}\right] D_0^d + \pi\left[\rho(1-\gamma)(1-\beta) - \left[\nu - \rho(1-\gamma)\right] Z)\right].$$

The first thing to stress is that no insurance is purchased if default is anticipated. To see that, note that, for $D_0^d = 1$,

$$\frac{\partial E((Y(1,Z))}{\partial Z} = \pi\left[\nu - \rho(1-\gamma)\right] > 0 \Longleftrightarrow \nu > \rho(1-\gamma),$$

whereas for $D_0^d < 1$,

$$\frac{\partial E((Y(D_0^d, Z))}{\partial Z} = \pi\left[\rho(1-\gamma) - \nu\right] - \pi\nu\frac{\left[\rho\left(1-\pi\right) - 1\right]}{1 - \left(1-\pi\right)\gamma\rho} < 0 \Longleftrightarrow \nu > \frac{1}{(1-\pi)} - \gamma\rho. \tag{46}$$

From (13) and (6), we know that $\upsilon > \rho > \frac{1}{(1-\pi)}$, so both conditions hold.

Then, substituting $Z = 0$ into (45),

$$D_0^d = 1 \Longleftrightarrow \overline{x} \geq \widetilde{x} \equiv \frac{1 - \left(1-\pi\right)\gamma\rho}{\left(1-\pi\right)\gamma}$$

Also, for the default option to be the equilibrium we need that

$$D_0^d = \min\left\{\frac{(1-\pi)\,\gamma\overline{x}}{1-(1-\pi)\,\gamma\rho};1\right\} > D_0^{nd} = \frac{\gamma\overline{x}+(1-\beta)\pi\nu}{1-\gamma\rho+\pi\nu} > \frac{\gamma\overline{x}}{1-\gamma\rho+\pi\nu},$$

which in turn implies that

$$\left.\frac{\partial D_0^d}{\partial\overline{x}}\right|_{\overline{x}<\widetilde{x}} = \frac{(1-\pi)\,\gamma}{1-(1-\pi)\,\gamma\rho} > \frac{\partial D_0^{nd}}{\partial\overline{x}} = \frac{\gamma\overline{x}}{1-\gamma\rho+\pi\nu} > \left.\frac{\partial D_0^d}{\partial\overline{x}}\right|_{\overline{x}=\widetilde{x}} = 0.$$

Therefore, to show that the country never chooses the default option, it suffices to show that it is so for $\overline{x}=\widetilde{x}$, the point at which the additional borrowing in period 0 relative to the non-default case is maximized.

Finally,

$$D_0^d\left(\overline{x}=\widetilde{x}\right) = 1 > D_0^{nd} = \frac{\gamma\widetilde{x}+(1-\beta)\pi\nu}{1-\gamma\rho+\pi\nu}.$$

which would imply

$$\nu > \frac{1}{(1-\pi)\,\beta},$$

which is never verified for small π, since $\nu < \frac{1}{\beta}$

7.2.4 Case 4I $\left(x_3^{IN} > \overline{x} \geq 0\right)$

If $\overline{x} < x_3^{IN}$, the country does not purchase insurance and we are back in the benchmark case. To ensure that no default occurs, it is sufficient to show that

$$\overline{x}_3^{IN} - \overline{x}_4^{B} = \frac{\pi(1-\beta)(1-\gamma)}{\gamma\left(\gamma\rho\,(1-\pi)+\pi\gamma-\pi-\gamma\right)} > 0$$

which is always the case for sufficiently small values of π.

7.3 Catastrophe lending facility

As before, for high initial incomes ($\overline{x} \geq x_1^M = \frac{1-\gamma(\rho-\beta)}{\gamma}$), the borrowing constraint (22) does not bind and the country invests the optimum in both states. In turn, for $\overline{x} < x_1^M$, from (23)

$$D_0^{nd} = \frac{\gamma\left(\overline{x}+1-\beta\right)}{1-\gamma\left(\rho-1\right)} \leq 1,$$

and

$$M = D_0^{nd} - (1-\beta) = \frac{\gamma\overline{x}-(1-\gamma\rho)\,(1-\beta)}{1-\gamma\,(\rho-1)} > 0 \Longleftrightarrow \overline{x} > x_4^M \equiv \frac{(1-\gamma\rho)\,(1-\beta)}{\gamma} = x_3^{IN}, \qquad (47)$$

Alternatively, a financially constrained country may choose to increase borrowing from private lenders (at a risk-adjusted rate $i = \frac{1}{1-\pi}$) at the expense of defaulting on bonds if hit by a shock. However, unlike in the benchmark, now the country would still have access to multilateral lending in period 2. In this scenario, period 0 borrowing D_0^d should be such that the country repays in good states:

$$D_0^d \leq (1-\pi)\,\gamma(\overline{x}+\rho D_0^d)$$

from which

$$D_0^d = \frac{(1-\pi)\,\gamma\overline{x}}{1-(1-\pi)\,\gamma\rho} \leq 1 \Longleftrightarrow \overline{x} \leq \widetilde{x}_2^M \equiv \frac{1-(1-\pi)\,\gamma\rho}{(1-\pi)\,\gamma},$$

with $\widetilde{x}_2^M \leq \overline{x}_1^M$ for small π, and

$$\begin{aligned} M^d &= D_0^d - (1-\beta) = \frac{(1-\pi)\,\gamma\overline{x}}{1-(1-\pi)\,\gamma\rho} - (1-\beta) \geq 0 \\ \iff & \overline{x} \geq \widetilde{x}_3^M \equiv (1-\beta)\left[\frac{\frac{1}{(1-\pi)} - \gamma\rho}{\gamma}\right] > x_4^M \end{aligned}$$

From (24) and (**??**), we know that the condition for a default on bonds is linear in $\overline{x}$:

$$\begin{aligned} \Delta\left(D_0^d, D_0^{nd}, M^d, M^{nd}\right) &\equiv E(Y(D_0^d, M^d) - E(Y(D^{nd}, M^{nd}) > 0 \\ &= (1-\pi)\left(\overline{x} + \rho D_0^d\right) + \pi\,(1-\gamma)\left[\overline{x} + \rho(1-\beta+M^d) - M^d\right] - D_0^d \qquad (48) \\ &> (1-\pi)\left(\overline{x} + \rho D_0^{nd}\right) + \pi\left[\overline{x} + \rho(1-\beta+M^{nd}) - M^{nd}\right] - D_0^{nd} \end{aligned}$$

and can be written as

$$\Delta\left(D_0^d, D_0^{nd}, M^d, M^{nd}\right) = D_0^d - D_0^{nd} - \frac{\pi}{(1-\pi)\rho - 1}\left\{\gamma\left[\overline{x} + \rho(1-\beta)\right] - (\rho-1)\left[(1-\gamma)\,M^d - M^{nd}\right]\right\}. \quad (49)$$

As before, we can distinguish three intervals:

- $\left[\widetilde{x}_2^M, x_1^M\right]$ where $D_0^d = 1 > D_0^{nd} = \frac{\gamma[\overline{x}+1-\beta]}{1-\gamma(\rho-1)}$,
- $\left[\widetilde{x}_3^M, \widetilde{x}_2^M\right]$ where $D_0^d = \frac{(1-\pi)\gamma\overline{x}}{1-(1-\pi)\gamma\rho}$; $D_0^{nd} = \frac{\gamma[\overline{x}+1-\beta]}{1-\gamma(\rho-1)} < 1$, $M^{nd} > 0$; $M^d > 0$,
- $\left[\overline{x}_4^M, \widetilde{x}_3^M\right]$ where $D_0^d = \frac{(1-\pi)\gamma\overline{x}}{1-(1-\pi)\gamma\rho} < 1-\beta < D_0^{nd} = \frac{\gamma[\overline{x}+1-\beta]}{1-\gamma(\rho-1)}$, and $M^{nd} > M^d = 0$, and no default occurs

Given the linearity of (49), to characterize the equilibrium it suffices to check the thresholds for the first two intervals.

Trivially, for $\overline{x} = x_1^M$, (49) does not hold and no default occurs, since $D_0^d = D_0^{nd} = 1$. In turn, for $\overline{x} = \widetilde{x}_2^M$, $D_0^d = 1 > D_0^{nd}$ implies that (49) always holds for small enough π. Finally, for $\overline{x} = \widetilde{x}_3^M$,

$$D_0^d\left(\widetilde{x}_3^M\right) = 1-\beta < \left[\frac{1}{(1-\pi)} + \rho\,(1-\gamma)\right](1-\beta) = D_0^{nd}\left(\widetilde{x}_3^M\right) \qquad (50)$$

and, again, the country does not default.

It follows that there is an interval $\left[x_3^M, x_2^M\right]$, such that $x_1^M > x_2^M > \widetilde{x}_2^M > x_3^M > \widetilde{x}_3^M > x_4^M$, within which the country borrows $D_0^d = \min\left\{\frac{(1-\pi)\gamma\overline{x}}{1-(1-\pi)\gamma\rho}, 1\right\}$ in period 0, and, if hit by the shock, borrows $M^d = D_0^d - (1-\beta)$ from the contingent loan credit and defaults.

The thresholds for the default interval are obtained directly from (49) as:

$$x_2^M \equiv \overline{x} : \Delta\left(1, D_0^{nd}, \beta, M^{nd}\right) = 0 = \frac{(\rho-1)-\pi}{\gamma\,(1-\pi\gamma)\,(\rho-1)} - (\rho-\beta)$$

and

$$x_3^M \equiv \overline{x} : \Delta\left(D_0^d, D_0^{nd}, M^d, M^{nd}\right) = 0 = \frac{(1-\beta)\,(1-\pi\gamma)\,(\rho-1)\left(1-\left(1-\pi^2\right)\gamma\rho\right)}{\pi^2\left(1+\gamma^2(\rho-1)\right) + \gamma\,(\rho-1) - \pi\left(\gamma^2(\rho-1)+\rho\right)}.$$

7.4 Catastrophe lending and insurance

For $x \in [x_1^M, x_2^{IN}]$, the country minimizes the amount of (costly) insurance such that it still attains $L_0 = 1$, and $L_1 = \beta$. The borrowing constraint (4) then becomes:

$$(D_0 + \pi\nu Z) \leq \gamma(\overline{x} + \rho((1-\beta) + Z + M)) - \gamma M, \tag{51}$$

which, substituting,

$$M = D_0 - (1-\beta) - Z$$

yields

$$D_0 \leq \frac{(\overline{x} + 1 - \beta + Z)\gamma - \pi\nu Z}{1 - \gamma(\rho - 1)} \equiv \widetilde{D}_0^{MIN} \tag{52}$$

Finally, we have that

$$\widetilde{D}_0^{MIN} \geq 1 \Longleftrightarrow Z \geq \widetilde{Z}^{MIN} \equiv \frac{1 - \gamma(\overline{x} - \beta - \rho)}{\gamma - \pi\nu},$$

and, in addition, for M to be non negative, we need

$$Z \leq \beta.$$

The two conditions are simultaneously verified for

$$\overline{x} \geq \frac{1 + \pi\beta\nu - \gamma\rho}{\gamma} = x_2^{IN}$$

Consider now the case $\overline{x} \leq x_2^{IN}$. We can now substitute D_0 from (51) in the expression for expected income, so that

$$E(Y) = (1-\pi)\left(\overline{x} + \rho D_0^{bl}\right) + \pi(\overline{x} + \rho(1 - \beta + M + Z) - M) - D_0^{bl} - \pi\nu Z.$$

and, differentiating with respect to Z, we get

$$\frac{\partial E(Y)}{\partial Z} = \frac{(\pi + \gamma - \pi(1 - \pi - \gamma)\nu)\rho - \gamma(1 + \pi\nu + \pi\rho^2)}{1 - \gamma(\rho - 1)},$$

which is always positive for small enough π. Thus for $\overline{x} \leq x_2^{IN}$ we are back to the insurance case.

7.5 A contingent catastrophe lending facility

The model can be readily modified to represent this case: we simply need to note that no multilateral assistance is forthcoming in the event of default ($M^d = 0$), which tilts the balance against the default decision: default, while still possible, is associated with narrower interval.

Under this new assumption, $M^d = 0$, and the borrowing constraint (22) becomes

$$\Delta\left(D_0^d, D_0^{nd}, 0, M^{nd}\right) = D_0^d - D_0^{nd} - \frac{\pi}{(1-\pi)\rho - 1}\left\{\gamma\left[\overline{x} + \rho(1-\beta)\right] + (\rho - 1)M^{nd}\right\}.$$

It is immediate to verify that, for $\overline{x} = \widetilde{x}_3^M$, $D_0^d = 1$,

$$D_0^{nd}\Big|_{\overline{x} = \widetilde{x}_2^M} = \frac{(1 - \gamma\rho + \gamma(1-\beta))}{1 - \gamma(\rho - 1)} < 1$$

and

$$\Delta\left(1, D_0^{nd}, M^{nd}\right) = 1 - D_0^{nd} - \frac{\pi}{(1-\pi)\rho - 1}\left\{\gamma\left[\overline{x} + \rho(1-\beta)\right] + (\rho - 1)M^{nd}\right\} > 0$$

for small π. On the other hand, for $\overline{x} = \widetilde{x}_3^M$, $M^d = 0$ and we are back in the previous case, where from (50) we know that the country chooses to borrow less and avoid default.

Thus, following the steps of the previous proof, it can be shown that there is an interval $\left[\overline{x}_2^{M'}, \overline{x}_3^{M'}\right]$ such that $x_1^M > \overline{x}_2^{M'} > \widetilde{x}_2^M > x_3^{M'} > \widetilde{x}_3^M > x_4^M$, within which the country borrows $D_0^d = \min\left\{\frac{(1-\pi)\gamma\overline{x}}{1-(1-\pi)\gamma\rho}, 1\right\}$ in period 0, and, if hit by the shock, defaults.

The thresholds of this interval are defined by the zeros of

$$\begin{aligned}\Delta\left(1, D_0^{nd}, M^{nd}\right) &= 1 - \frac{\gamma\left(\overline{x}+1-\beta\right)}{1-\gamma\left(\rho-1\right)}\left[1+\frac{\pi}{(1-\pi)\rho-1}\left(\rho-1\right)\right] \\ &\quad -\frac{\pi}{(1-\pi)\rho-1}\left\{\gamma\left[\overline{x}+\rho(1-\beta)\right]-\left(\rho-1\right)\left(1-\beta\right)\right\}\end{aligned}$$

and

$$\begin{aligned}\Delta\left(D_0^d, D_0^{nd}, M^{nd}\right) &= \frac{\left(1-\pi\right)\gamma\overline{x}}{1-\left(1-\pi\right)\gamma\rho} - \frac{\gamma\left(\overline{x}+1-\beta\right)}{1-\gamma\left(\rho-1\right)}\left[1+\frac{\pi}{(1-\pi)\rho-1}\left(\rho-1\right)\right] \\ &\quad -\frac{\pi}{(1-\pi)\rho-1}\left\{\gamma\left[\overline{x}+\rho(1-\beta)\right]-\left(\rho-1\right)\left(1-\beta\right)\right\},\end{aligned}$$

from which

$$x_2^{M'} = \frac{\left(\rho-1\right)\left(1+\beta\gamma-\gamma\rho\right)-\pi\left(1+\left(\rho-1\right)\left(\beta-\left(\beta\left(1-\gamma\right)+\gamma\right)\gamma\rho\right)\right)}{\gamma\left(\rho-1\right)\left(1-\pi\gamma\right)}$$

and

$$x_3^{M'} = \frac{\left(\rho-1\right)\left(1-\beta\right)\left(1-\left(1-\pi\right)\gamma\rho\right)\left(\gamma+\pi\left(1-\gamma\right)\gamma\rho-\pi\right)}{\gamma\left(\pi+\gamma\left(\rho-1\right)-\pi\left(2-\gamma\right)\left(1+\gamma\right)\rho+\pi^2\left(1-\left(1-\gamma\right)\gamma\left(\rho-1\right)\right)\rho+\pi\left(1-\gamma\right)\gamma\rho^2\right)}.$$

Figure 1

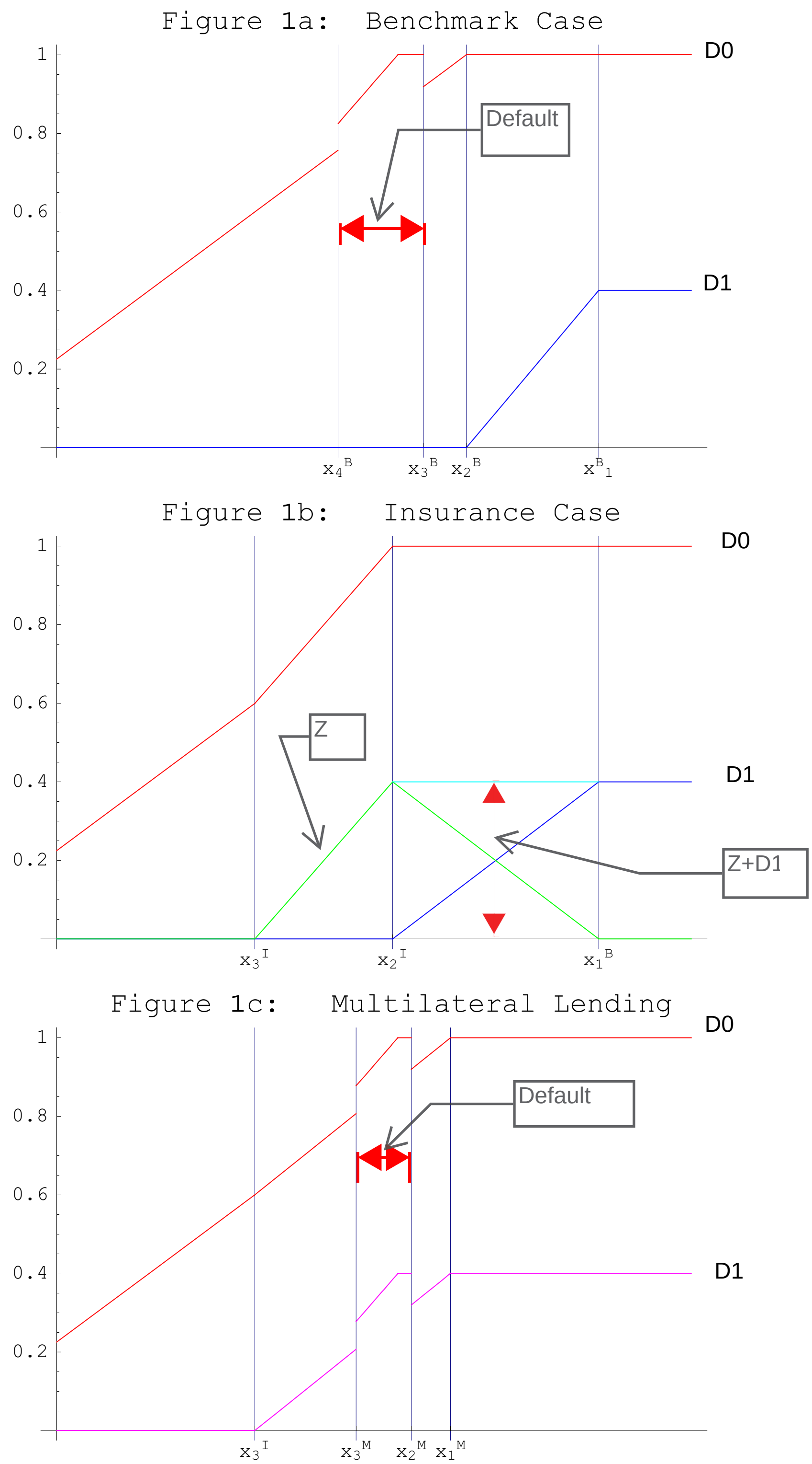

Figure 1a: Benchmark Case
D0
D1
Default
1
0.8
0.6
0.4
0.2
x_4^B
x_3^B
x_2^B
x^B_1
Figure 1b: Insurance Case
D0
D1
Z
Z+D1
x_3^I
x_2^I
x_1^B
Figure 1c: Multilateral Lending
D0
D1
Default
x_3^I
x_3^M
x_2^M
x_1^M

Figure 2

Fig. 2a: Net Income in Good and Bad States(Red=B,Green=I, Blue=M)

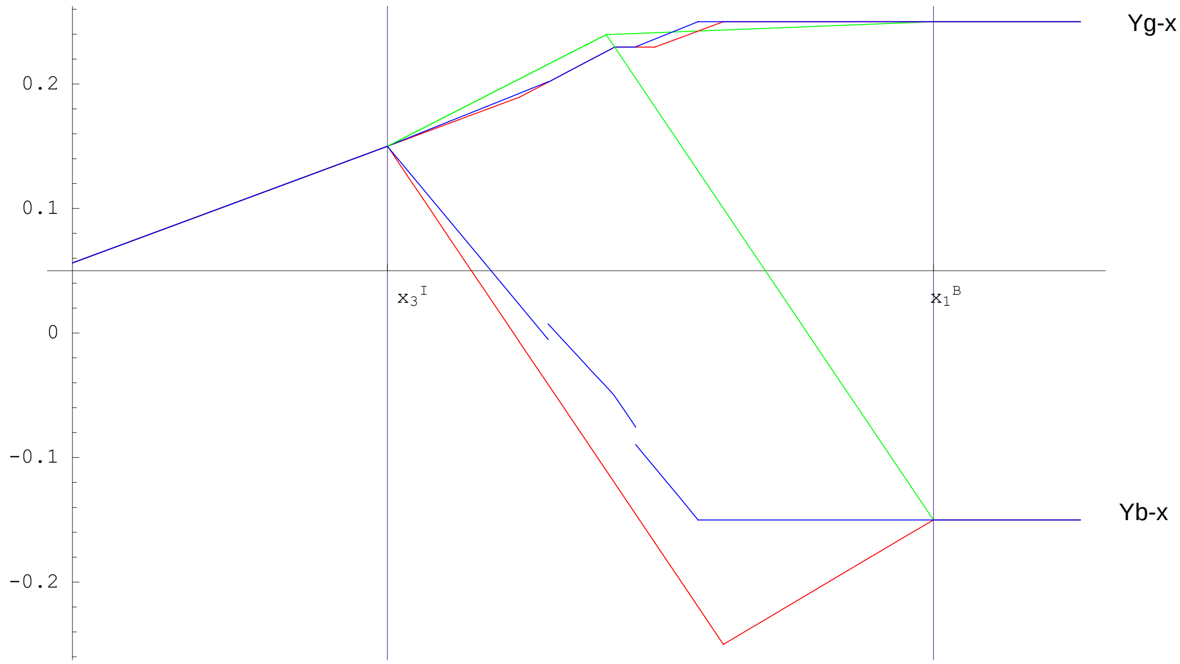

Fig. 2b: Expected(Net) Income (Red=B, Green=I, Blue=M)

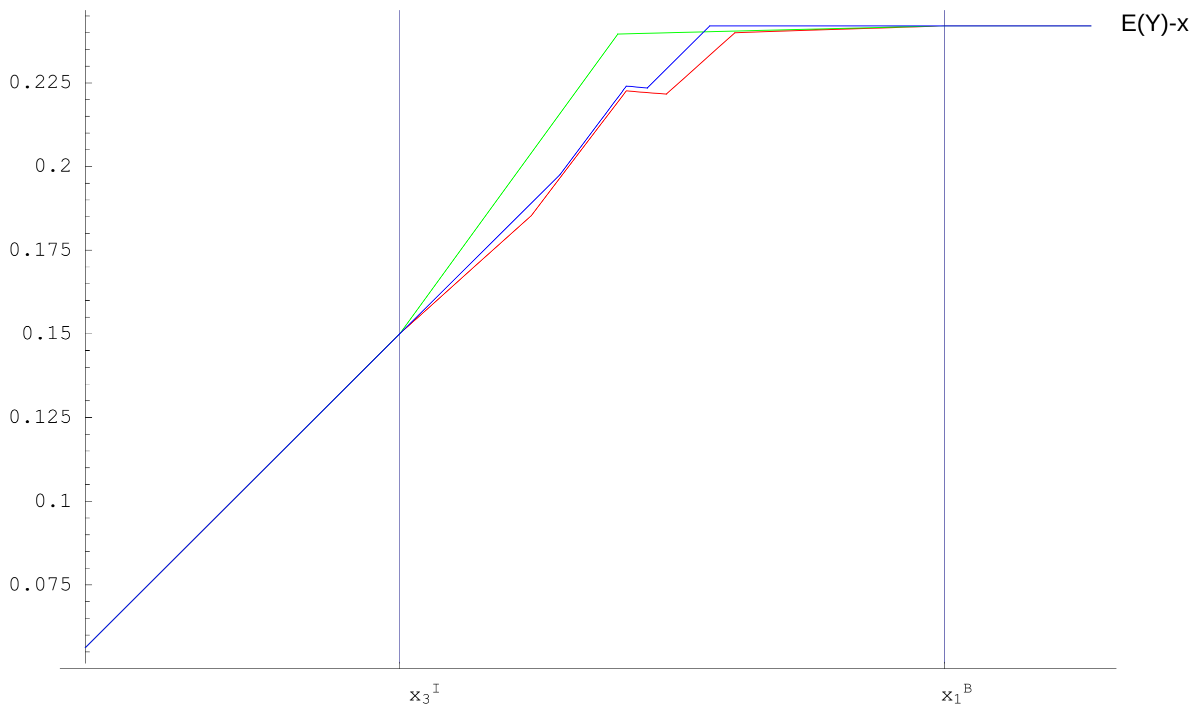

Figure 3

Fig. 3a: Mult. Lending and Insurance(MI)

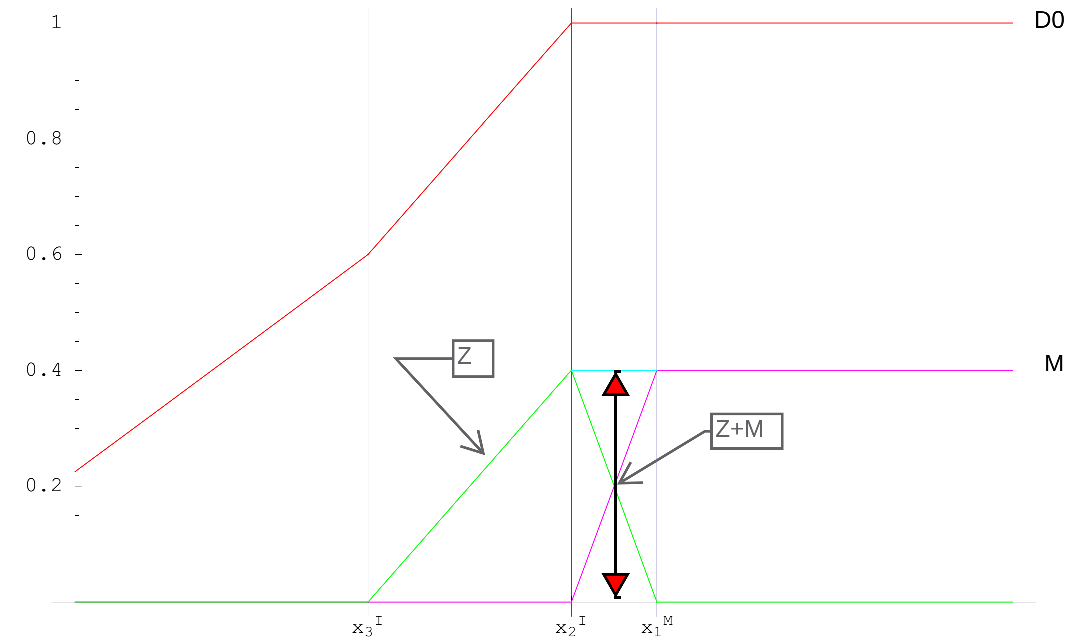

Fig. 3b: Expected(Net) Income (Red=B, Green=IN, Orange=MI)

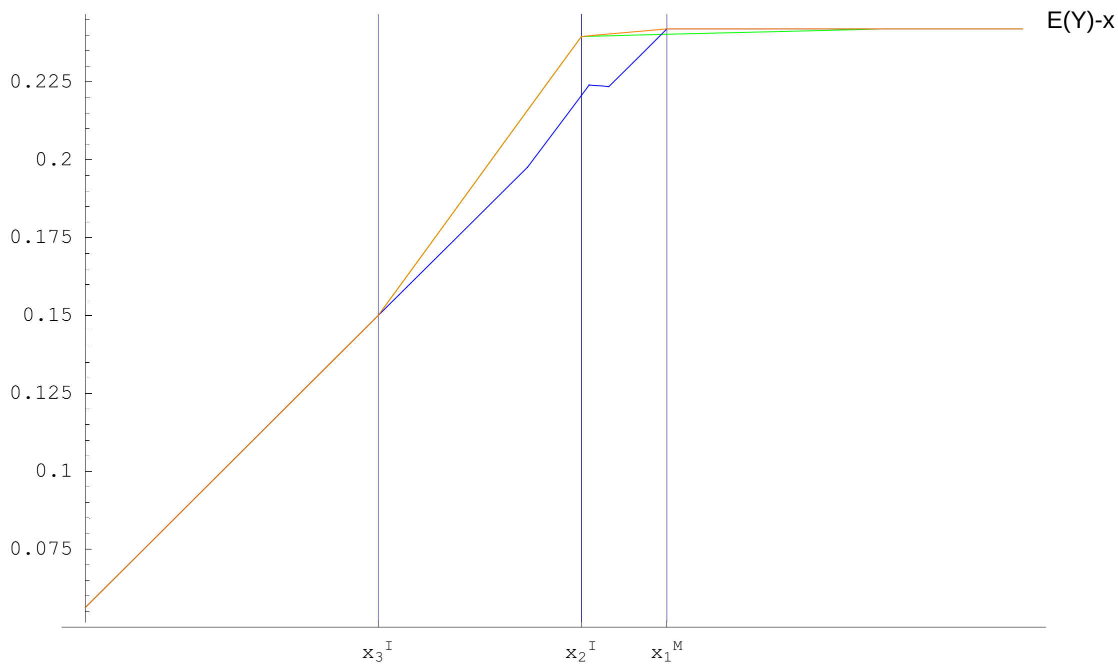

Fig.4: Exp. (Net) Income (Blue= Mult.Facil., Yellow= Mult. Cont. Facil.)