MULTILATERAL INDEXED LOANS AND DEBT SUSTAINABILITY

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Abstract

We evaluate the benefits and costs of indexing multilateral loans to variables related to developing countries' ability to pay and, thus, whether a reform of multilateral lending is feasible and economically justified. The analysis covers 40 IDA countries from 1990 to 2010 and focuses on three types of debt: GDP-indexed loans; export-indexed loans; inflation-indexed loans denominated in local currency. The insurance that indexed debt might offer against macroeconomic shocks depends on the conditional covariances of GDP growth, real exchange-rate depreciation and net exports that we estimate as the covariances of the forecast errors obtained from a VAR model. The analysis shows that GDP-indexed loans would help to stabilize the debt ratio of the majority of IDA countries in our sample but a larger number of them would benefit from loans denominated in local currencies. The cost of such reform would be small or non-existent. The estimation of a CAPM suggests that loans indexed to GDP or denominated in local currencies could be introduced at current interest rates since the estimated risk premium is less than one percent. Any additional risk for multilateral lenders would be more than offset by a lower frequency of debt crises.

JEL Classification: F34, F37, G11, H63.

Keywords: Debt sustainability, low income countries, export-indexed debt, GDP-indexed debt, local currency debt, multilateral loans, risk insurance, VAR forecasting.

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1. Introduction

It is commonly held that GDP indexation, by introducing contingencies in sovereign debt, may help stabilize the debt-to-GDP ratio, reduce the likelihood of debt crises and sovereign defaults, and limit the pro-cyclicality of fiscal policy (see e.g. Shiller, 1993; Borensztein and Mauro, 2004; Griffith-Jones and Sharma, 2006). The idea is that indexed debt can provide valuable insurance since its payments are linked to the underlying conditions of the borrower, notably those that impact on its ability to pay. While debt instruments can either be indexed to GDP, exports or commodity prices, the key feature is that they imply lower payments in times of output contractions, export shortfalls or terms of trade shocks, that is, precisely when a country struggles to honor its debt.

In this paper we evaluate the benefits and costs of indexed loans provided by Multilateral Development Banks (MDBs) to low income countries (LICs) and thus whether a reform of multilateral lending is feasible and economically justified. To this end, we provide new evidence for a group of 40 IDA countries over the 1990-2010 period for three types of debt: i) foreign currency loans indexed to real GDP; ii) foreign currency loans indexed to the dollar value of exports; iii) inflation-indexed loans denominated in local currency.

We investigate the role of indexed loans in reducing IDA countries' vulnerability to adverse macroeconomic shocks in a model where indexed debt may help stabilize the debt ratio and thus reduce the likelihood of a debt crisis. The optimal type of indexation depends on the conditional variances and covariances of GDP growth, real exchange-rate depreciation and net exports that we estimate as the covariances of the forecast errors obtained from a VAR model of each IDA country. In doing so, we pay particular attention to the currency denomination of debt obligations, an issue so far neglected in the literature on GDP indexation, studying to what extent GDP- or export-indexed loans can also reduce debt vulnerability to exchange rate movements. Finally, to assess the cost for multilateral lenders of a program of indexed lending, we examine the potential for risk diversification, studying the risk-return characteristics of a portfolio of indexed loans to IDA countries.

Portfolio risk analysis shows that individual country risk could be easily diversified, since the volatility of the MDBs' portfolio is much lower than the average volatility of individual loans for all types of indexation considered. However, the estimation of a Capital Asset Pricing Model (CAPM), where OECD growth is taken as the relevant market-portfolio return, suggests that the risk of export-indexed loans is difficult to hedge because IDA countries' exports are strongly correlated with OECD growth and thus with the fiscal resources of multilateral lenders. By contrast, loans indexed to GDP or denominated in local currencies could be introduced at current interest rates because their risk premium is less than one percent and more than offset by the lower risk of debt distress.

The estimation of the optimal shares of indexed loans provides strong evidence in favor of domestic currency loans indexed to inflation and some support to GDP indexation. Lending in the borrower's currency helps to stabilize the debt ratio against unanticipated movements in the real exchange rate which are a main cause of debt vulnerability. We also find supportive evidence for GDP-indexed loans, but while such instruments provide valuable insurance to a majority of IDA borrowers in our sample, they benefit a fewer number of countries than domestic currency loans.

After this Introduction, Section 2 reviews the literature on GDP-indexed debt and discusses the benefits of indexation. Section 3 presents preliminary evidence on macroeconomic risk affecting LICs. A simple model is derived in Section 4 where indexed loans help to stabilize the debt ratio. The optimal shares of GDP-indexed debt, export-indexed debt and domestic currency debt depend on the conditional covariances of debt returns with GDP, net exports and the real exchange-rate which are estimated in Section 5 as the covariances of the forecast errors of individual country VAR models. Section 6 evaluates the cost of indexation for multilateral lenders by investigating the potential for portfolio diversification. Section 7 discusses the arguments against indexation and Section 8 concludes.

2. The benefits of indexed debt; Literature review

Contingent debt contracts fall into two categories. The first type of indexed debt has payments linked to an exogenous variable, that is, one out of the debtor country's control. Relevant examples of exogenous variables are the price of a country's export commodity or the growth rate of industrialized countries. A strong case for commodity-indexed debt was first made by Besley and Powell (1989) who called for intervention by multilateral institutions in developing a market for such debt.[1]

The second type of indexed debt has payments linked to an endogenous variable which is at least partly under the control of the debtor country. A first proposal in this direction can be dated back to the work of Bailey (1983) who argued for converting the external debt into proportional claims on exports. Other possible endogenous indices are the country's total output or its trade balance. Lessard (1987) and Helpman (1989) contend that some sort of output indexation might be beneficial as a risk-sharing or hedging mechanism: risk-averse countries could shift some of their exposure to better diversified lenders.

Krugman (1988), Froot et al. (1989) and Kletzer et al. (1992) considered the relative merits of indexing debt to exogenous versus endogenous variables. This literature took for granted the insurance benefits of debt linked to variables that proxy for the country's ability to pay; it did not investigate whether a greater insurance could be provided by indexing to commodity prices or GDP or exports, and focused instead on the incentive effects and moral-hazard costs of indexation. For instance, Krugman (1988) argued in favor of commodity-price indexation to minimize the risk of moral hazard implied by the debtor's ability to affect exports.

Although commodity-indexed debt may be as good an instrument for insurance and risk sharing as GDP- or export-indexed debt, in this paper we focus on the latter types of indexation for two reasons. First, while a few experiments have been made with commodity-price indexation, much less is known regarding GDP indexation despite the Argentine experiment and a growing literature on the benefits (and pricing) of GDP-indexed bonds.[2] Second, we are interested in studying the potential for a reform of multilateral lending that could find the largest possible application and not be confined to specific export producers.

Interest in GDP indexation has been revived by the work of Shiller (1993, 2003, 2004, 2005) and Borensztein and Mauro (2004). With the aim to improve international risk sharing, Shiller proposes to create 'macro markets' for GDP-linked securities, taking the form of perpetual claims on a fraction of a country's GDP. Borensztein and Mauro argue for the introduction of bonds with coupon payments augmented by the issuing country's GDP growth rate to reduce cyclical vulnerability and the probability of debt crises. In their view, GDP-indexed bonds should provide insurance against output contractions and act as an automatic stabilizer thus allowing for greater flexibility in fiscal policy.[3]

In fact, indexed debt would provide debtor countries and their citizens with insurance against shocks affecting their income, net exports and tax revenues.[4] In particular, GDP-indexed debt

[1] More recently, recommendations for emerging countries to issue commodity-indexed debt have been made by Haldane (1999), Daniel (2001), Caballero (2003a, 2003b) and Atta-Mensah (2004).

[2] See Griffith-Jones and Sharma (2006) and Costa et al. (2008) for a review of the Argentine and other experiences with GDP indexation.

[3] See Obstfeld and Peri (1998) and Drèze (2000a) for a discussion of the stabilization role of GDP-indexed bonds in advanced economies.

[4] The literature on debt management also stresses that GDP-indexed debt minimizes tax-rate adjustments by providing a hedge against shocks to the tax base (see e.g. Barro 1995 and Missale 1997).

stabilizes the debt-to-GDP ratio against output contractions and slow growth while export-indexed debt limits the accumulation of external debt due to terms-of-trade shocks. Hence, contingent debt makes a country's debt position resilient to adverse shocks and enhances its sustainability. By linking debt payments to the borrower's ability to pay, debt either indexed to GDP or exports reduces the likelihood of debt crisis and default (see, e.g., Borensztein and Mauro 2004). In addition, to the extent that indexation provides automatic relief to countries in distress, it avoids the loss of value associated with prolonged debt crises and delays in debt restructuring.

Indexed debt also makes fiscal policy less procyclical by acting as an 'automatic-stabilizer'; it reduces the need for fiscal adjustment in bad times, when output or exports are lower than expected, and forces fiscal moderation in good times when GDP or exports are unexpectedly high. Finally, indexed debt can benefit the poor in that it reduces pressure to cut spending for social programs in bad times (Griffith-Jones and Sharma 2006). As a built-in mechanism for macroeconomic stability, indexed debt is also beneficial for growth.

The case for GDP indexation has been put forth by Griffith-Jones and Sharma (2006) and Kamstra and Shiller (2010), while issuers' and investors' concerns have been addressed in two studies by the UN (2005, 2006) with the aim to find workable solutions and define a strategy for implementation. In fact, the introduction of GDP-indexed bonds is difficult. A first problem is the delay with which estimates of GDP become available and their later, sometimes substantial, revisions. A second problem is the complexity of the instrument that makes its pricing difficult. As a result, the empirical literature on pricing GDP-indexed bonds has grown fast (see e.g. Kruse et al. 2005, Pernice and Fagundez 2005, Chamon and Mauro 2006, Costa et al. 2008, Ruban et al. 2008).

Pricing difficulties are instead not an obstacle for indexing non-marketable loans as those provided by MDBs to LICs. This makes the introduction of indexed loans by MDBs a more realistic project whose chances of success are worth investigating. Moreover, as macroeconomic stability and debt reduction are main goals of development assistance, indexed loans would bring specific benefits to MDBs that have not been considered in the literature so far. In particular, multilateral lenders would gain from a lower risk of debt distress and a lower frequency of debt crises since indexation would reduce the likelihood that debtor countries run into repayment difficulties and eventually file for debt relief. Indeed, providing explicit insurance against macroeconomic shocks can be more effective than debt relief to deal with repayment difficulties because it avoids delays in delivering assistance and saves on the costs associated with debt restructuring.

Another advantage of making debt payments contingent on economic performance is that multilateral lenders need not decide in advance which countries are worth receiving large loans and which should instead obtain small grants, as it currently happens for IDA assistance under the Debt Sustainability Framework (DSF). As poor growth or export performance would reduce payments on indexed loans and make them similar to grants, indexed loans could also be extended to countries where debt sustainability is considered at risk.

A few papers exist, closely related to our analysis, which examine the potential for a reform of multilateral lending. Tabova (2005) proposes that MDBs make loans to LICs with amortization payments conditional on the growth rate of GDP. In particular, she examines the effectiveness of a scheme that partially exempts countries from debt service when growth is lower than expected using historical simulations of debt service to IDA. Drèze (2000b) suggests the use of GDP-indexed bonds (with a deductible) as part of a strategy to restructure the debt of the poorest countries. However, there is no general agreement that GDP-indexed loans are superior to other debt instruments. Guillaumont et al. (2003) and Cohen et al. (2007) contend that MDBs should extend loans to LICs with amortization payments indexed to the value of exports. On the contrary, Hausmann and Rigobon (2003) and Levy Yeyati (2007) argue that MDBs should provide inflation-indexed loans denominated in local currencies to reduce developing countries' exposure to real exchange-rate movements which are a main cause of debt vulnerability.[5]

[5] According to Levy Yeyati local currency loans to emerging countries could be funded by multilateral securities

In this paper we compare three types of multilateral indexed loans that have been considered in the literature: i) foreign currency loans indexed to real GDP in local currency units; ii) foreign currency loans indexed to the dollar value of exports; iii) local currency loans indexed to inflation.

3. The need for insurance: Stylized facts

A natural starting point to study the benefits of indexed loans, and their role in stabilizing the debt-to-GDP ratio, is to look at the volatilities of GDP, exports and the real exchange rate. As sudden changes in such variables may determine large variations in the debt ratio, this evidence gives some preliminary indication of the need for insurance.

Table 1 shows the standard deviations of the annual growth rates of real GDP in local currency, of the dollar value of exports, and of the real exchange rate (more precisely, the dollar deflator) for 64 IDA countries and 31 high income OECD countries over the period 1990-2010.[6] The volatility of GDP growth for the IDA group is 5.4 percent while it is only 2.9 percent for OECD economies. Although it is well known that growth in developing countries is more unstable, the volatility of GDP growth in IDA countries is sizeable; it is almost two times that of industrial countries. Interestingly, the volatility of growth and thus the need for insurance increase with debt, as shown by a 0.53 correlation with the debt ratio in 2000.

Export growth is even more volatile; its standard deviation in the IDA group is 20.8 percent, three times higher than the 6.7 percent exhibited by OECD countries. This result does not depend on the presence of outliers, as shown by the maximum standard deviation and a median volatility only slightly lower than the average. Since exports are the main source of foreign exchange needed to service the external debt, their uncertainty exposes IDA countries to the risk of debt distress. However, export volatility is only weakly correlated with the debt ratio.

IDA countries are also exposed to exchange-rate risk because of the foreign currency denomination of their debts. In fact, their capacity to pay depends, not on the value of their GDP in local currency, but on their GDP in current US dollars. This implies that changes in the dollar deflator of GDP, say, the real exchange rate, are destabilizing; their wealth effects are a major cause of debt vulnerability and crises. Table 1 shows that the rate of depreciation of the real exchange rate in IDA countries is 10 times more volatile than in OECD economies. Since this evidence may depend on few episodes of hyper-devaluations, we computed the average standard deviation excluding three outliers.[7] Despite this correction, the average volatility of the real exchange rate remains high at 17.6 percent, twice as large as in OECD economies. Highly indebted countries thus appear to be particularly exposed to real exchange-rate risk, since its correlation with the debt ratio is almost 0.5. Hence, not only the real exchange rate matters for debt service, but it also tends to be significantly more volatile at high levels of debt. A possible objection is that the GDP in current US dollars is a more relevant indicator of a country's ability to pay than its exchange rate. However, Table 1 shows that the volatility of dollar GDP remains sizeable at 14.9 percent, similar to that of the real exchange rate, even after excluding countries with hyper-devaluation episodes.

We have so far examined the uncertainty of GDP, exports and the real exchange rate over one year which is a very short time horizon from the perspective of debt sustainability. In fact, it can be argued that the annual volatilities of such variables do not matter for LICs' ability to pay since the debt they owe to multilateral creditors is very long term. To shed some light in this issue, Table 2 reports the average growth rates of GDP and exports, and the rate of depreciation of the real exchange rate of IDA countries for two periods: 1990-2000 and 2000-2010. The result is striking:

denominated in the same currencies that would attract the demand of domestic residents for credit risk-free assets.

[6] All data are from the World Bank World Development Indicators database except for data on exports that are taken from the UNCTAD database.

[7] We excluded Angola, Congo Democratic Republic and Nicaragua.

for most countries there is a huge difference between the two periods. Viewed in a long term perspective, the uncertainty surrounding LICs' ability to pay is even worse than what the annual volatilities suggest. The performance of such countries over a long future horizon is very difficult to predict on the basis of their historical experience, which raises doubts on the reliability of the Debt Sustainability Analysis (DSA) as a way to decide which countries are worth receiving large loans and which should instead obtain small grants, as it currently happens for IDA assistance under the DSF. Indeed, LICs are particularly vulnerable to macroeconomic shocks which play a major role in determining their debt sustainability. Then, the analysis should focus on the unanticipated variability of GDP, exports and the real exchange rate over a long future horizon, as we do in what follows.

4. A model of indexed debt and debt sustainability

In this section we present a simple model where indexed loans help to stabilize the debt ratio against adverse shocks to output, net exports and the exchange rate, and thus reduce the likelihood of a debt crisis.

To ensure the sustainability of the external debt the LIC and the MDB implement the necessary actions and lending strategies to maintain the debt-to-GDP ratio within a threshold limit that, according to the DSF, varies across countries depending on the quality of their policies and institutions. To this end, a strategy is decided taking into account debt repayments and the possible realizations of output, inflation, the exchange rate, net exports, etc., as outlined in stress test scenarios. The outcome of these efforts is however uncertain since unforeseeable events, i.e. bad shocks, Z , can still put the external debt on an unsustainable path. A debt crisis materializes if the debt-to-GDP ratio, B_{t+1}, exceeds the threshold level B^T:

$$B_{t+1} = \hat{B}_{t+1} + Z > B^T \tag{1}$$

Where $\hat{B}_{t+1}$ is the debt ratio before the shock, and Z is a shock to the current account or net transfers or FDI or the realization of contingent liabilities that occur after the stabilization program has been carried out.

The the LIC and the MDB minimize the probability that the debt ratio exceeds the threshold, B^T:

$$min\, E_t Prob\left[Z > B^T - \hat{B}_{t+1}\right] = min\, E_t \int_{B^T-\hat{B}_{t+1}}^{\infty} \Phi(Z)dz \tag{2}$$

where $\Phi(Z)$ is the probability density function of Z, and E_t denotes expectations conditional on the information at time *t*.

The dynamics of the debt depends on the type of loans, on whether they are indexed or not, denominated in foreign or domestic currency. In the case of conventional loans denominated in US dollars and bearing a fixed interest rate, r, the debt ratio evolves as follows:

$$B_{t+1} = \hat{B}_{t+1} + Z = (1+r)\frac{S_{t+1}P_t Y_t}{S_t P_{t+1} Y_{t+1}} B_t - nx_{t+1} + Z \tag{3}$$

where S_t, is the nominal exchange rate (the price of foreign currency in domestic units); P_t, is the domestic price level; Y_t is the real GDP in domestic currency, and; nx_{t+1} is the ratio of net exports to GDP.[8]

Consider, now, loans indexed to real GDP, Y_t and denominated in foreign currency. We focus on

[8] Shock to net transfers and FDI are included in *Z* under the assumption that they are not systematically correlated with the other variables in the model, while their deterministic components are set to zero for simplicity.

capital-indexed loans which have all payments indexed to the ratio of real GDP, Y_t to its baseline (expected) level, Y_t^*, because they are superior, as insurance instruments, to interest-indexed loans.[9] In particular, capital indexation protects LICs against a permanent fall in GDP because the debt service is linked to the level of GDP and thus to the borrower's ability to pay contrary to interest-indexed loans whose payments depend on GDP growth which might turn quite high in a recovery from a recession while in fact the GDP has decreased since the date of issuance. More importantly, while debt service payments (interests and amortizations) on capital-indexed loans are a continuous function of GDP, the interest payments on interest-indexed loans cannot be negative which makes them insensitive to large negative realizations of GDP growth, that is, exactly when relief is most valuable.[10]

If all the debt is indexed to GDP, its ratio evolves as:

$$B_{t+1} = \hat{B}_{t+1} + Z = (1 + r^{\gamma}) \frac{S_{t+1} P_t Y_t}{S_t P_{t+1} Y_{t+1}} B_t \frac{Y_{t+1}}{Y_{t+1}^*} - nx_{t+1} + Z \tag{4}$$

Where r^{γ} is the fixed rate component of the interest rate, possibly different from r.

If we assume a baseline GDP equal to expected GDP, i.e. $Y_{t+1}^* = E_t Y_{t+1}$, and both types of debt, conventional and GDP-indexed, are outstanding, then the debt ratio can be linearly approximated as

$$B_{t+1} = (1 + r + e_{t+1} - \pi_{t+1} - g_{t+1})(1-\gamma)B_t + (1 + r^{\gamma} + e_{t+1} - \pi_{t+1} - E_t g_{t+1})\gamma B_t - nx_{t+1} + Z \tag{5}$$

where γ is the share of GDP-indexed debt chosen in period t, e_{t+1} is the rate of depreciation of the exchange rate, π_{t+1} is the inflation rate, g_{t+1} is the growth rate of real GDP, and nx_{t+1} is the ratio of net exports to GDP.

It is worth noting that, even if the debt were fully indexed to GDP, i.e. $\gamma = 1$, debt vulnerability would still arise from real exchange-rate depreciations, more precisely, from negative shocks to the dollar GDP deflator, $\pi_{t+1} - e_{t+1}$.

4.1 GDP-indexed versus conventional loan

To minimize the probability that the debt ratio exceeds its sustainable level, B^T, the LIC and the MDB choose the share of GDP-indexed loans, γ, that minimizes the probability (2) subject to equation (5). The first order condition of this problem is:

$$E_t\left[(r^{\gamma} - r + g_{t+1} - E_t g_{t+1})\Phi\left(B^T - \hat{B}_{t+1}\right)\right] = 0 \tag{6}$$

where $r^{\gamma} - r$ is the risk premium on GDP-indexed loans, $B^T - \hat{B}_{t+1}$ is the distance of the debt ratio from its sustainable level when $Z = 0$, and $\Phi\left(B^T - \hat{B}_{t+1}\right)$ is a function of γ.

The opportunity cost of GDP-indexed debt is equal to the sum of the risk premium, $r^{\gamma} - r$, and the unanticipated GDP growth, $g_{t+1} - E_t g_{t+1}$.[11] The first order condition (6) shows that the debt

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[9] Capital indexed loans have the principal linked to the level of GDP relative to its baseline level (i.e. its expected level) so that all payments are adjusted to the accumulated growth of GDP (relative to its baseline growth) from the time of issuance to the payment date. Interest indexed loans have, instead, interest payments determined by a floating rate that, in the simplest version, is the sum of a fixed coupon and a variable component equal to the difference between the growth rate of GDP and a baseline growth rate. A minimum zero rate rules out negative interest payments.

[10] The zero minimum rate makes interest indexation particularly unsuited for 38-year IDA loans which pay a service charge of only 0.75%, and have a back-loaded repayment profile because of an initial grace period of 6 years.

[11] It is worth noting that equation (6) assumes that the baseline growth coincides with expected growth and that the MDB and the LIC share the same expectations. If, for example, the LIC expected a higher growth rate, $E_t^{LIC} g_{t+1} > E_t g_{t+1}$, the perceived opportunity cost of servicing GDP-indexed debt would increase.

composition is optimal only if the opportunity cost of GDP-indexed lending is uncorrelated with the increase in the probability that debt stabilization fails. If this were not the case, this probability could be reduced by changing the debt structure; e.g. by substituting GDP-indexed loans for conventional loans or vice-versa.[12]

To derive an explicit solution for the optimal share, γ^*, of GDP-indexed debt we must specify the density function $\Phi(Z)$. As the latter cannot be estimated, we take its linear approximation over the range of bad realizations, $Z > 0$, of the shock.[13,14] This implies a triangular density equal to

$$\Phi(Z) = \frac{\bar{Z} - Z}{\bar{Z}^2} \tag{7}$$

where $\bar{Z}$ is the worst possible realization of the shock to debt accumulation.

Substituting equations (5) and (7) in the first order condition (6) yields the optimal share of GDP-indexed debt:

$$\gamma^* = \frac{Var(g_{t+1})}{Var(g_{t+1}) + (r^\gamma - r)^2} - \frac{Cov\big(g_{t+1}(e_{t+1} - \pi_{t+1})\big)}{Var(g_{t+1}) + (r^\gamma - r)^2} + \\ + \frac{Cov(g_{t+1} nx_{t+1})}{[Var(g_{t+1}) + (r^\gamma - r)^2]B_t} - (r^\gamma - r)\frac{\big[\bar{Z} - \big(B^T - E_t B_{t+1}^{\gamma=0}\big)\big]}{[Var(g_{t+1}) + (r^\gamma - r)^2]B_t} \tag{8}$$

where $Var(.)$ and $Cov(.)$ denote variances and covariances conditional on the information available at time t, and $E_t B_{t+1}^{\gamma=0}$ is the expected debt ratio when γ is equal to zero.

The optimal share, γ^*, depends on both risk and cost considerations. GDP-indexed debt is a perfect hedge against variations in the debt ratio due to output fluctuations but, to stabilize the debt, it must provide insurance against real exchange-rate risk and net exports uncertainty. This is the case if GDP growth is lower than expected when the real exchange rate depreciates so that the capital loss on dollar denominated debt is hedged by lower payments on indexed loans. A positive covariance of GDP growth with net exports makes indexed loans an even better hedge as it implies lower debt payments when net exports are unexpectedly low. The case for indexation weakens as the conditional variance of GDP growth increases, thus producing unnecessary fluctuations in interest payments. The last term in equation (8) shows that the optimal share of GDP-indexed debt decreases with the premium, $r^\gamma - r > 0$, that the MDB asks on indexed loans. As discussed in Section 6, this premium could however be small to the extent that GDP risk is uncorrelated across countries and thus diversifiable in the MDB's portfolio.

4.2 Export-indexed versus conventional loans

Export-indexed loans advocated by Guillaumont et al. (2003) may offer an alternative to GDP indexation since exports are correlated to output and less subject to misreporting. In what follows, we consider loans with payments indexed to the deviation of the dollar value of exports from its baseline (expected) level.

Defining with v_{t+1} the nominal growth rate of exports, the payments on export-indexed loans can be linearly approximated by $r^x + v_{t+1} - E_t v_{t+1}$, where the expected growth rate, $E_t v_{t+1}$, is assumed to be equal to the baseline growth rate.[15]

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[12] The argument assumes that there are non-negative constraints to the choice of debt instruments.

[13] The triangular density is the linear approximation of any density function decreasing with Z for $Z > 0$. It implies that bad realizations of the shock, Z, are less likely to occur the greater is their size.

[14] We assume that the strategy in the DSA is expected to stabilize the debt, so that $B^T > \hat{B}_{t+1}$.

[15] Assuming capital indexation, and denoting exports as X_t, the return on indexed debt is equal to $(1 + r^x)B_t \frac{X_{t+1}}{X_{t+1}^*}$.

If both types of loans, conventional and export-indexed, are outstanding, the debt ratio is equal to

$$B_{t+1} = (1 + r + e_{t+1} - \pi_{t+1} - g_{t+1})(1 - x)B_t + \\ +(1 + r^x + v_{t+1} - E_t v_{t+1} + e_{t+1} - \pi_{t+1} - g_{t+1})xB_t - nx_{t+1} + Z \quad (9)$$

where x is the share of export-indexed debt.

The LIC and the MDB choose x to minimize the probability (2) that the debt ratio exceeds its sustainable level subject to equation (9). The first order condition of this problem is:

$$E_t\big[(r^x - r + v_{t+1} - E_t v_{t+1})\Phi\big(B^T - \hat{B}_{t+1}\big)\big] = 0 \quad (10)$$

where $r^x - r$ is the risk premium on export-indexed loans, $B^T - \hat{B}_{t+1}$, is the distance of the debt ratio from its sustainable level when $Z = 0$, and $\Phi(B^T - \hat{B}_{t+1})$ is a function of x.

The first order condition (10) shows that the debt composition is optimal only if the opportunity cost of additional export-indexed lending is uncorrelated with the increase in the probability that debt stabilization fails.

To derive an explicit solution for the optimal share, x^*, we rely on the triangular probability density function (7) that linearly approximates $\Phi(Z)$ over the range $Z > 0$.

Substituting equations (7) and (9) in the first order condition (10) yields the optimal share of export-indexed debt:

$$x^* = \frac{Cov(v_{t+1} g_{t+1})}{Var(v_{t+1}) + (r^x - r)^2} - \frac{Cov\big(v_{t+1}(e_{t+1} - \pi_{t+1})\big)}{Var(v_{t+1}) + (r^x - r)^2} + \\ + \frac{Cov(v_{t+1} nx_{t+1})}{[Var(v_{t+1}) + (r^x - r)^2]B_t} - (r^x - r)\frac{[\bar{Z} - (B^T - E_t B_{t+1}^{x=0})]}{[Var(v_{t+1}) + (r^x - r)^2]B_t} \quad (11)$$

where $Var(.)$ and $Cov(.)$ denote variances and covariances conditional on the information available at time$t$, and $E_t B_{t+1}^{x=0}$ is the expected debt ratio when x is equal to zero.

The optimal share, x^*, depends on both risk and cost considerations. Export-indexed debt is clearly not as good a hedge against output fluctuations as GDP-indexed debt, but it may provide a better insurance against a real depreciation or a fall in net exports. This would be the case if unanticipated export growth were positively correlated to net exports and real appreciations. While the first condition is likely to hold, the relation between export growth and the exchange rate is uncertain and may depend on the country considered. For instance, if export growth is driven by productivity improvements or terms of trade effects which lead to a real appreciation, then export indexation helps to stabilize the debt ratio. On the other hand, if export growth results from a real depreciation driven by capital outflows, then indexation introduces an additional source of instability. The case for indexation also depends on the conditional variance of export growth; a high volatility of exports reduces the benefits of insurance since it leads to unnecessary fluctuations in interest payments. Finally, the optimal share of indexed debt decreases with the premium, $r^x - r$, asked on such loans.

4.3 Domestic currency loans versus conventional loans

The foreign currency denomination of LICs' debt is a major source of vulnerability. Hausmann and Rigobon (2003) and Levy Yeyati (2007) have argued that MDBs should lend to developing countries in local currencies to insulate their debts from exchange rate movements. They also stress that domestic currency loans should be indexed to the price level to eliminate inflationary temptations and offer partial protection to multilateral lenders. In what follows we compare the performance of domestic currency loans to that of conventional loans in stabilizing the debt ratio.

The nominal value of the credit outstanding on inflation-indexed loans denominated in local currency is revalued each period at the rate of inflation so that the domestic currency debt accumulates, in nominal terms, at the rate $r^h + \pi_{t+1}$, where r^h is the real interest rate on inflation-indexed loans.

With both types of debt, conventional and inflation-indexed debt denominated in local currency, the debt ratio evolves as

$$B_{t+1} = (1 + r + e_{t+1} - \pi_{t+1} - g_{t+1})(1-h)B_t + (1 + r^h - g_{t+1})hB_t - nx_{t+1} + Z \qquad (12)$$

where h is the share of inflation-indexed debt denominated in local currency.

The LIC and the MDB choose h to minimize the probability (2) subject to equation (12), which yields the following first order condition:

$$E_t\left[(r^h - r + \pi_{t+1} - e_{t+1})\Phi\left(B^T - \hat{B}_{t+1}\right)\right] = 0 \qquad (13)$$

It is worth noting that the interest rate differential, $r^h - r$, is not equal to the risk premium which is instead given by the expected return differential (in foreign currency), $r^h + E_t\pi_{t+1} - E_t e_{t+1} - r$. Defining $\hat{r}^h \equiv r^h + E_t\pi_{t+1} - E_t e_{t+1}$, the first order condition (13) can be written as:

$$E_t[\hat{r}^h - r + (\pi_{t+1} - E_t\pi_{t+1}) - (e_{t+1} - E_t e_{t+1})]\Phi\left(B^T - \hat{B}_{t+1}\right) = 0 \qquad (14)$$

which has the usual interpretation, and where $\Phi(B^T - \hat{B}_{t+1})$ is a function of h.

Using the triangular probability density function (7) and equation (12) in the first order condition (14) yields the optimal share of domestic currency debt:

$$h^* = \frac{Var(e_{t+1} - \pi_{t+1})}{Var(e_{t+1} - \pi_{t+1}) + (\hat{r}^h - r)^2} - \frac{Cov\left((e_{t+1} - \pi_{t+1})g_{t+1}\right)}{Var(e_{t+1} - \pi_{t+1}) + (\hat{r}^h - r)^2} + \\ - \frac{Cov\left((e_{t+1} - \pi_{t+1})nx_{t+1}\right)}{[Var(e_{t+1} - \pi_{t+1}) + (\hat{r}^h - r)^2]B_t} - (\hat{r}^h - r)\frac{\left[\bar{Z} - \left(B^T - E_t B_{t+1}^{h=0}\right)\right]}{[Var(e_{t+1} - \pi_{t+1}) + (\hat{r}^h - r)^2]B_t} \qquad (15)$$

where $Var(.)$ and $Cov(.)$ denote variances and covariances conditional on the information available at time t, and $E_t B_{t+1}^{h=0}$ is the expected debt ratio when the h is equal to zero.

Inflation-indexed debt denominated in local currency stabilizes the debt against movements in the real exchange-rate, but is not the best hedge against all types of shocks. For instance, dollar denominated debt would fare better against an export shortfall due to a real appreciation. On the other hand, a negative covariance between GDP growth and real depreciation strengthens the case for domestic currency loans. Hence, the latter would mostly benefit countries experiencing sudden stops and currency crises characterized by real depreciations and output contractions. Finally, the optimal share of local currency debt decreases with the expected return differential, $\hat{r}^h - r$.

If the MDBs were willing to extend the concessionality of conventional loans to domestic currency loans, the real interest rate should be equal to $r^h = r + E_t e_{t+1} - E_t\pi_{t+1}$ so as to equalize the expected returns on the two loans. If the real exchange rate is expected to appreciate, this requires a negative real interest rate, r^h, given the low rate on conventional loans, r. As the interest rate cannot be negative, the same result could be accomplished through a proper reduction of amortization payments proportional to the expected real appreciation, which is the equivalent of discounting payments by the cumulative growth of a baseline expected real exchange rate.

5. The stabilizing debt structure: An empirical assessment

In this section, we empirically investigate whether loans indexed either to GDP, exports or inflation, may help to stabilize the debt-to-GDP ratio against macroeconomic shocks and thus enhance debt sustainability in LICs. To evaluate the insurance benefits of indexed loans we must estimate the conditional variances and covariances of their returns with GDP growth, real exchange-rate depreciation and net exports (relative to GDP). In what follows we present the econometric strategy to estimate the relevant conditional variances and covariances that can be used in equations (8), (11) and (15) to calculate the optimal share of each type of indexed debt under the assumption that MDBs ask on indexed loans the same interest rate (service charge) as on conventional loans; i.e. that risk premia, $(r^{\gamma} - r)$, $(r^{x} - r)$, $(\hat{r}^{h} - r)$ are all equal to zero.

5.1 Conditional variances and covariances: A VAR approach

The theoretical model presented in Section 4 derives the optimal shares of indexed loans as functions of the variances and covariances of GDP growth, real exchange-rate depreciation and net exports over a long future horizon conditional on the information available at time t. More precisely, these variances and covariances represent the conditional second moments of the unanticipated components of future realizations of such variables.

Our empirical strategy is to recover the theoretical conditional variances and covariances from the estimation of the variances and covariances of forecast errors obtained from a vector autoregression (VAR) analysis of the relations between GDP growth, real exchange-rate depreciation and net exports. This approach appears particularly suitable in the present case for a number of reasons. First, the forecast errors capture both the deviation of the variable realization from the conditional mean (the forecast) and the error made in the forecast, for example, in estimating baseline GDP growth. Second, we are interested in the projected, out-of-sample, relations between the unanticipated components of the relevant variables. Third, we are interested in estimating such relations over a long future horizon because IDA loans have a long maturity, currently of 38 years. Fourth, this approach is consistent with DSA stress tests, but, unlike the latter that consider a combination of shocks, it captures the stochastic relations between shocks as estimated from VAR analysis.

The strategy consists in estimating, for each country, a VAR model for the following variables: the log of the real GDP (in local currency units), the log of the real exchange rate (i.e. the log of the reciprocal of the dollar deflator), the log of the dollar value of exports, and the ratio of net exports to GDP. A set of exogenous variables is also included to improve the forecasting ability of the model: the lagged US long-term interest rate, the log of the real GDP of OECD countries in the previous year, and the lagged debt-to-GDP ratio, as well as a constant term and a linear trend. All data are from the World Bank World Development Indicators database except for data on exports that are taken from the UNCTAD database. More details on the econometric model are reported in the Technical Appendix.

5.2 Estimation results

To evaluate whether indexed loans would provide insurance against macroeconomic shocks and stabilize the debt ratio of LICs, we first assume that MDBs would not require a risk premium on indexed loans, and set $(r^{\gamma} - r)$, $(r^{x} - r)$ and $(\hat{r}^{h} - r)$ equal to zero in equations (8), (11) and (15). Conditional on indexed loans having the same expected return as conventional loans, we can compute the optimal shares of indexed debt and shed some light on the extent of hedging that the three types of indexation can offer.[16] We shall investigate the feasibility of zero risk premia in Section 6 where we examine the risk of indexed lending.

[16] Note that, besides risk premia, computing the optimal shares would require heroic assumptions about $\bar{Z}$ and $E_t B_{t+1}$.

We estimate the conditional variances and covariances in equations (8), (11) and (15) using the covariances of the forecast errors (at the 5-year and 10-year horizons) obtained from the VAR model described in the previous section that is estimated for each IDA country over the period from 1990 to 2010.[17] The estimated conditional covariances relative to conditional variances are reported in Tables A1 and A2 in the Appendix.

Tables 3 and 4 show the optimal shares of indexed debt at the 5-year and 10-year horizon, respectively, for the 40 IDA countries (which have data on exports to estimate the VAR[18]). Each table reports: the debt ratio in Column 2; the risk of debt distress in Column 3; the ratio of the variance of exports to the variance of GDP in Column 4; the ratio of the variance of real exchange rate depreciation to the variance of GDP in Column 5; the optimal share, γ^*, of debt indexed to real GDP in Column 6; the optimal share, x^*, of debt indexed to the dollar value of exports in Column 7, and; the optimal share, h^*, of inflation-indexed debt denominated in local currency in Column 8.

Despite the relatively low volatility of GDP growth, GDP-indexed loans would be a valuable hedge for many IDA countries. The optimal share, γ^*, is positive in 29 out of 40 countries at both horizons considered, and greater than one in 25 and 28 countries at the 5- and 10-year horizons, respectively. Hence, in the latter countries, GDP indexation does not only help to stabilize the debt ratio but provides additional insurance. The reason is that GDP growth is lower than expected when the real exchange rate depreciates and thus when capital losses realize because of foreign currency denomination. In fact, equation (8) shows that the optimal share of GDP-indexed debt is greater than one if GDP growth either displays a negative covariance with exchange-rate depreciation or a positive covariance with net exports. A closer look at the estimated covariances in Tables A1 and A2 clearly points to the negative relation between output growth and the exchange rate as the main reason for the additional insurance that GDP indexation provides. This evidence is consistent with the Balassa-Samuelson effect where productivity shocks lead to higher growth and unexpected real appreciations. Sudden stops and capital outflows also lead to output contractions and large real depreciations. Hence, in bad times, GDP indexation can provide insurance against valuation effects, a benefit not yet considered in the literature.

However, GDP-indexed loans are not always the best instrument; as shown by a negative γ^* in one fourth of the countries in our sample loans should not be indexed to GDP. The reason of this result is a significant negative covariance of GDP growth with net exports with, besides a positive relation with real depreciation. In fact, if imports increase with GDP, then higher payments on indexed loans add to the accumulation of external debt. Finally, the case for GDP-indexed loans appears to be unrelated to the risk of debt distress, since countries are equally distributed in all risk categories.

Export indexation is a valid alternative to GDP indexation. Tables 3 and 4 show that some export-indexed loans would help stabilize the debt ratio in 34 IDA countries at both horizons considered. The reason why export-indexed loans provide a hedge against output fluctuations is that exports are correlated with GDP. This positive relation may derive from either productivity improvements or export diversification or foreign demand. Interestingly, export indexation, if optimal, also provides a hedge against valuation effects due to real depreciations. Though consistent with the effects of productivity improvements, this result more likely reflects the impact of terms-of-trade shocks on the value of exports and the real exchange rate. It suggests that export-indexed loans could provide an alternative to commodity price indexation, especially to those countries that do not depend on a single commodity export.

However, we find 4 and 6 countries that would not benefit from export indexation over the 5-year and 10-year horizon, respectively.[19] At the longer horizon, there are other 8 countries for which

[17] The optimal number of lags for both the endogenous and exogenous variables has been determined by minimizing the Bayesian Information Criterion (BIC).

[18] Table 2 shows that 25 IDA countries do not have sufficient data on exports for VAR estimation.

[19] It is worth noting that in 5 countries conventional loans are preferable to both GDP and export indexation.

indexation should cover less than half of the debt. In general, a low or negative share of indexed loans is due to the positive covariance between export growth and exchange-rate depreciation but in few cases it results from a significant negative covariance between export growth and the ratio of net exports to GDP. Though surprising, this evidence can be explained by the positive relation characterizing the dynamics of exports, GDP and the demand for foreign goods.

The last instruments we consider are inflation-indexed loans denominated in the borrower's currency. Local currency loans provide full protection against real exchange-rate depreciation and appear to stabilize the debt ratio in a large number of countries. The last columns of Tables 3 and 4 show that 36 countries would benefit from having at least a fraction of their debts denominated in local currency and indexed to inflation. The optimal share of such debt is equal or greater than 0.9 in 29 countries at the 5-year horizon and in 26 countries at the 10-year horizon. As noted earlier, in most countries, the covariance between exchange-rate depreciation and GDP growth is negative. This implies an additional insurance role for domestic currency debt; it avoids the capital loss from currency depreciation when this is most valuable, i.e. when output growth is lower than expected. Indeed, real exchange-rate depreciations due to productivity shocks and capital outflows are a major cause of debt vulnerability which calls for either indexing the debt to GDP or denominating it in local currency. On the other hand, domestic currency loans are not useful to hedge against shocks to net exports. We find that in about half of our sample the ratio of net exports to GDP is positively correlated to exchange-rate depreciation which explains why, in most cases, the optimal share of inflation-indexed debt denominated in local currency is close to one.

The optimal share of domestic currency debt is negative in only five countries, either at the 5- or 10-year horizon: Djibouti, Malawi, Mongolia, Solomon Islands and Vanuatu. In these countries exposure to currency risk allows to hedge changes in net exports because the latter are particularly sensitive to exchange-rate depreciation. Interestingly, conventional loans are the best instruments for Djibouti and Vanuatu where no indexation scheme appears to enhance debt sustainability.

The analysis provides strong evidence in favor of domestic currency lending and inflation indexation. This policy seems also easier to implement because inflation indexation has been widely experimented and inflation is released with fewer lags and less revisions than either data on GDP or exports. We also find supportive evidence for GDP-indexed and export-indexed loans, but though such instruments provide valuable insurance to the majority of IDA countries in our sample, they benefit fewer countries than local currency loans. A main lesson from our analysis is that a 'one size fits all solution' does not exist to the problem of stabilizing the debt ratio.

6. The cost of insurance

In this section we assess the feasibility of extending indexed loans at the same interest rate as that on conventional loans by studying whether portfolio diversification can limit the risk exposure of multilateral lenders. To this end we examine the risk-return characteristics of a portfolio of indexed loans to the same group of IDA countries considered in the previous section. First, we provide evidence on 10-year returns that arise because of forecast errors in setting the baseline levels of the reference variables. Secondly, we compare the return volatility of the MDBs' portfolio to the volatility of individual country loans. Finally, we derive the risk premium (over conventional loans) of indexed-loan portfolios through CAPM beta coefficients estimated using the covariances of the forecast errors of individual country VAR models of indexed loan returns.

6.1 Risk diversification; Return and volatility of a portfolio of indexed loans

Indexed loans offer borrowers valuable insurance in case long-run GDP, exports or the real exchange rate turn out to be lower than expected. Indeed, payments on indexed loans would be linked to deviations of real GDP, exports and the real exchange rate from their baseline expected levels.[20] If the trends of such variables were perfectly foreseen, indexed loans would still offer a hedge against cyclical or exchange-rate fluctuations, and help stabilize the budget but, as the ability to pay in the long run remains the same, the need for protection would be limited. As forecast errors of long-term GDP, exports and the real exchange rate are usually large, so is the scope for insurance and likewise large are the potential gains and losses that MDBs would experience on individual country loans. Then, the relevant issue is whether there is enough heterogeneity across LICs that positive and negative returns would average out in the MDBs' portfolio.

To address this issue we simulate the performance of a portfolio of indexed loans to IDA countries over the period 2000-10 where the country weights in the portfolio are equal to their fraction of multilateral debt at the end of 2000. We first compute the returns on individual country loans and, then, the MBDs' portfolio returns for the three types of indexation. Returns depend on the realization of real GDP (or exports or the real exchange rate) in 2010 relative to its baseline level in the same year as expected and set in 2000. To this end, we assume that expectations would be half based on historical values and half based on perfect foresight and simply set the expected growth rates of each reference variable equal to the mean of the growth rates observed in the two periods 1990-2000 and 2000-2010. Then, we compute the unexpected 10-year return on each country loan as the difference between the cumulative growth of GDP (or export or the real exchange rate) and the cumulative growth of baseline GDP over the period 2000-2010. Finally, we estimate (for each type of indexation) the 10-year return on the MDBs' portfolio by taking the weighted average of the 10-year returns on individual country loans.

Figure 1 shows the 10-year returns (relative to conventional loans) for GDP-indexed, export-indexed and inflation-indexed domestic-currency loans, from the top to the bottom panel. The continuous line displays, in increasing order, the returns on the individual country loans that arise because of forecast errors in setting the baseline levels of real GDP, exports, and the real exchange rate. The horizontal continuous line reports the 10-year return on the MDBs' portfolio, while the horizontal dashed line indicates the sample average of individual returns.

The return on the MDBs' portfolio is lower than the average of the returns on the individual country loans and is positive for all types of indexation considered. This reflects the improved performance of IDA countries over the last ten years relative to the previous decade; on average GDP and exports grew faster and real appreciation was stronger. Individual loans with negative returns are suggestive of the insurance that countries would obtain from indexed debt, while returns on the MDBs' portfolios provide evidence on the potential gains or losses that multilateral lenders would obtain from indexation.

Table 5 shows that the 10-year return on the portfolio of GDP-indexed loans is 9.7 percent, i.e. less than 1 percent a year, while the returns on individual loans vary from a minimum of −11.2 percent to a maximum of 108.9 percent. In fact, GDP forecast errors appear to be balanced with 20 countries lying in the 2 to 20 percent range and 9 countries displaying a worse performance than expected. By contrast, export indexed loans show a much higher volatility as errors in forecasting exports, and thus 10-year returns, range from −37.1 percent to an astonishing 369.4 percent. The 10-year return on the MDBs' portfolio is as high as 31.1 percent, due to 15 countries displaying returns greater than 50 percent against only 8 countries reporting a negative performance. The 10-year dollar return on the portfolio of inflation-indexed loans in local currencies is also high at 39.6 percent, i.e. almost 4 percent a year. Compared to export-indexed loans, the variation in individual loan returns is lower, from −35.2 to 144.6 percent, but returns greater than 50 percent are still observed in 17 countries while they are negative in only 6 countries.

While this evidence is clearly dependent on the particularly good performance of IDA countries in the 2000s, GDP indexation still appears less risky than export indexation and domestic currency

[20] Inflation-indexed loan contracts in local currency may not feature a baseline real exchange rate, but it is reasonable to assume that their degree of concessionaly would have to be adjusted to compensate IDA countries for the expected real appreciation of their exchange rates.

lending. In fact, GDP growth may be relatively easier to forecast as also shown by the greater balance of positive and negative errors. In any case, the argument that indexed loans would have to pay a risk premium over conventional loans finds no support in our sample; if anything, introducing a cap on indexation, that limits positive payments, would be advisable.

To further examine the scope for risk diversification we look at the volatility of annual returns on the same portfolios of indexed loans over the period 2000-10. In particular, we investigate whether the MDBs' portfolio would be less volatile than the individual country loans, as it would be the case if GDP (or export or the real exchange rate) were uncorrelated across countries.

For each IDA country in our sample we compute the series of yearly returns as the difference between the actual rates of GDP growth (or export growth, or real appreciation) and the corresponding expected growth rates as previously estimated by the mean of the growth rates observed in the two decades. Then, for each year, we compute the return on the MDBs' portfolio as the weighted average of the returns on the 40 country loans and take the standard deviation of the resulting series as the volatility of the MDBs' portfolio.

The results are reported in Figure 2 for GDP-indexed, export-indexed, and domestic-currency inflation-indexed loans, from the top to the bottom panel. The continuous line displays, in increasing order, the standard deviations of returns on individual country loans. The average standard deviation in the sample is indicated by the horizontal dashed line, while the horizontal continuous line reports the standard deviation of returns on the MDBs' portfolio. The visual impression is that the extent of risk diversification by lending to IDA countries is enormous whatever is the type of indexation. Indeed, the volatility of the MDBs' portfolio is not only significantly lower than the sample average of the volatilities of the 40 IDA loans, but very few countries display a return volatility lower than the MDBs' portfolio: just 5 for each type of indexation.

6.2 A Capital Asset Pricing Model of indexed loans

We use the CAPM to estimate the risk premium that lenders would require to index their loans. The CAPM implies an expected return on an indexed loan equal to

$$E_t R_{i,t+1} = R_{F,t+1} + \left[E_t R_{M,t+1} - R_{F,t+1}\right] \frac{Cov_t\left(R_{i,t+1}; R_{M,t+1}\right)}{Var_t\left(R_{M,t+1}\right)} \tag{16}$$

where $R_{i,t}$ is the return on the indexed loan, $R_{F,t}$ is the risk-free rate and $R_{M,t}$ is the return on the market portfolio (see Borensztein and Mauro 2004).

Hence, the risk premium depends on the systematic component of GDP risk (or export risk or real exchange-rate risk) that is captured by the conditional correlation of the loan return with the return on the market portfolio; i.e. the beta coefficient. The latter conveys information on whether the risk of the loan can be diversified away; the lower the beta coefficient the lower the premium.

As multilateral lenders do not hold other financial assets than country loans, to estimate the beta coefficients we take the GDP growth of OECD countries as the relevant market-portfolio return. This choice is justified by the fact that multilateral lenders, such as the IDA window of the World Bank, are mainly funded through the contributions provided by their wealthier member states. The higher the correlation of IDA countries' GDP growth (or export growth or real exchange-rate appreciation) with GDP growth in OECD countries —i.e. the higher the beta coefficients— the greater the risk that loan repayments may fall at times when fiscal resources for multilateral funding are scarce.

Denoting with I_t the variable to which loan payments are indexed, and abstracting from valuation effects, the return on an indexed loan can be approximated by $r^j + i_{t+1} - i^*_{t+1}$ where r^j is the fixed-rate component of the return (with $j = \gamma, x, h$), while i_{t+1} is the log of I_{+1t} and i^*_{t+1} is the log of its baseline level. Then, defining with w_t the log of GDP in OECD countries, and noting that r^j, i^*_{t+1} and w_t and are known at t, the beta coefficient is equal to

$$\beta_{CAPM} = \frac{Cov_t\left[\left(r^i + i_{t+1} - i^*_{t+1}\right); \left(w_{t+1} - w_t\right)\right]}{Var_t[w_{t+1} - w_t]} = \frac{Cov_t[i_{t+1}; w_{t+1}]}{Var_t[w_{t+1}]} \tag{17}$$

To estimate the beta coefficients we use the variances and covariances of the forecast errors of the VAR model presented in section 5.1.[21] This allows us to derive beta coefficients over investment horizons of 5 and 10 years, which are particularly relevant because of the long maturity of multilateral loans.

Table 6 reports beta coefficients for: GDP-indexed loans; export-indexed loans; inflation-indexed loans denominated in local currency. They are computed as the covariances of the forecast errors of the log of GDP, exports and the real exchange rate with the log of GDP in the OECD area divided by the variance of the forecast error of the latter variable. The last rows of Table 6 also display beta coefficients for two MDBs' portfolios of indexed loans to IDA countries, along with their annual risk premium; the first portfolio, 'Multilateral Debt', uses as country weights their fraction of multilateral debt at the end of 2010; the second portfolio, 'Disbursement', takes as country weights the fraction of multilateral loan disbursement at the end of 2010.

In the case of GDP-indexed loans, beta coefficients are generally low for all countries at both horizons considered; indeed, a value greater than 0.5 is found only in 11 countries. The average of beta coefficients is 0.20 while the median is slightly higher at 0.28 for the 5-year horizon but decreases to 0.19 as the horizon extends to 10 years. This result clearly suggests that unanticipated GDP growth and thus the payments on indexed loans are, on average, very weakly correlated with OECD growth and thus with the fiscal resources of multilateral lenders.

The risk premium on GDP-indexed loans can be computed using equation (16). Taking the average growth rate of nominal GDP in the OECD area over the sample period, 4.6%, as the annual expected return on the market portfolio, and 0.75% as the risk-free interest rate on conventional IDA loans, the difference between their 10-year compounded returns is 0.49 = (0.568–0.078) and the 10-year compounded risk premium for the IDA country with the average beta, 0.20, is equal to 0.98 (= 0.49*0.20) which implies an annual risk premium lower than 1 percent. Even lower annual risk premia are shown in Table 6 for the two MDBs' portfolios; 67 and 89 basis points for the 'Multilateral Debt' portfolio and 19 and 34 basis points for the 'Disbursement' portfolio at the 5-year and 10-year horizon, respectively. In fact, beta coefficients, and thus output growth, vary considerably across countries, which suggests, once again, large opportunities for risk diversification.

Export-indexed loans expose multilateral lenders to much greater risk. As shown in Table 6, beta coefficients are positive and large with an average around 2.0 at both horizons considered. This finding cannot be attributed to a few outliers, since the beta coefficient for the median IDA country is higher at 2.23 and 2.61 for the 5-year and 10-year horizon, respectively. Indeed, there are 24 countries with a coefficient greater than 0.5. This evidence suggests that export growth is fairly correlated with OECD growth and thus with the fiscal resources of multilateral lenders. As a result, export-indexed loans would expose multilateral lenders to the risk of lower reflows at times when their funding needs are highest because of the weak performance of OECD donors. Table 6 shows that the annual risk premium for the two MDBs' portfolios would be very high; 880 and 909 basis points for the 'Multilateral Debt' portfolio and 809 and 889 basis points for the 'Disbursement' portfolio at the 5-year and 10-year horizon, respectively. As a 8 percent interest rate on multilateral loans is hard to imagine, we should conclude that export indexation is not a feasible option.

A different conclusion is reached if we consider inflation-indexed loans denominated in local currencies. Table 6 shows that the dollar return on domestic currency loans tends to be negatively

[21] The VAR model is extended to include an equation for the log of GDP of OECD countries..

correlated with OECD growth, as shown by the negative average and median beta coefficients at both horizons considered. The beta coefficient for the median IDA country is –0.27 at the 5-year horizon and decreases to –0.63 at the 10-year horizon. Since the real exchange rate of IDA countries tends to appreciate when OECD growth is lower than expected, the capital gains on local currency loans may stabilize donors' resources. This is the case for 24 countries in our sample for which the risk premium implied by the CAPM is even negative. When the two portfolios are considered, a positive risk premium of 62 and 143 basis points for the 'Multilateral debt' and 'Disbursement' portfolio, respectively, is found at the 5-year horizon but disappears at the 10-year horizon over which the risk exposure of multilateral lenders should be evaluated. Therefore, it appears that no premium over conventional loans should be asked on domestic-currency loans, a result which is consistent with the findings of Hausmann and Rigobon (2003).

The analysis suggests that the individual country risk could be easily diversified in the MDBs' portfolio in the case of GDP-indexed and local currency loans while the risk of export-indexed loans would be difficult to manage. Summing up, there are ample opportunities for risk-sharing among IDA countries, making it feasible for MDBs to provide loans either indexed to GDP or to inflation and denominated in local currencies. Such loans could be extended at current interest rates, since the estimated risk premium is either less than 1 percent or non-existent.

Although it appears that multilateral lenders could easily diversify GDP and exchange-rate risks, this result hinges on the assumption that GDP indexation or domestic currency lending would not eventually alter the behavior or the policies of the debtor countries. We turn to this issue in the next section.

7. The arguments against indexation

Loans indexed to variables that are partly under the borrower's control may give rise to adverse incentives and moral hazard. In the case of GDP indexation, a debtor government may behave opportunistically along three dimensions. First, GDP-indexed debt, by reducing the risk of distress and the probability of default, may favor irresponsible fiscal policies and delay fiscal adjustment. Secondly, the government might have less incentive to adopt policies that promote growth. Finally, the debtor country might be tempted to manipulate GDP data in order to pay less on its debt.

Insurance strategies naturally involve a trade-off between the benefits of insurance and the risk of moral hazard whose seriousness is difficult to assess. Many studies contend that the relevance of moral hazard is exaggerated (see e.g. Griffith-Jones and Sharma 2006, UN 2004, 2005). In particular, it is unlikely that debtor governments would ever take deliberate actions to forestall growth that is an important policy target and crucial to attract foreign investments and other capital flows. If anything, conventional fixed-rate debt does not seem to be effective as a disciplinary device, as shown by the long history of debt defaults, restructuring and relief. Moreover, disincentive effects may also emerge with conventional debt because of 'debt overhang'; a heavy debt burden may act as an implicit tax on future income, and reduce incentives for investment and policy reforms.

The low quality of LICs' national accounts is instead a serious issue for GDP indexation whether or not it leads to misreporting. Despite the continuous improvement of measurement standards and the substantial efforts by international institutions in checking for data consistency, an indexation program would require further advances in the reliability and verification of national accounts.

In light of the data requirements for GDP indexation, one may wonder whether inflation-indexed lending in local currencies would not provide a simpler solution to protect IDA countries from ouput contractions that are often associated with strong real depreciations. Unfortunately, local currency debt also raises incentive issues that cannot be fully solved by price indexation. For instance, indexation may not remove the temptation to implement a monetary expansion and depreciate the currency if this policy does not lead to higher inflation, as it would be the case in the presence of price controls. Moreover, while price inflation takes time to build up, the exchange rate usually moves fast and overshoots its long run level. To the extent that nominal exchange-rate depreciation exceeds inflation, multilateral lenders are not protected against the devaluation effects of monetary expansions. This may explain why they have been so far reluctant to denominate their loans in local currencies.

Summing up, indexed debt may give rise to moral hazard; local currency denomination may lead to greater laxity in monetary policy whereas GDP indexation, and the low quality of GDP data, may favor misreporting. In addition, the lack of reliable price indexes and the low quality of GDP data are practical obstacles to both types of indexation. While the relevance of these problems is difficult to assess, we have shown that, for most IDA countries, the insurance benefits of contingent debt would be substantial.

Finally, it is worth noting that arguments against GDP-indexed bonds, that they are difficult to price, illiquid and a costly financial innovation, do not apply to multilateral loans which are not traded and do not need a market. On the other hand, the non-marketability of loans introduces practical difficulties of a different kind that have not yet been noted in the literature, but are a serious obstacle to indexed lending. In fact, for GDP indexation to work, a baseline trend for GDP must be specified in the loan contract. If this were not the case, indexed loans would attract only countries with poor growth prospects. A similar problem arises in the case of loans denominated in local currency. This is because the real exchange rate of LICs not only tends to appreciate in the long run, but the expected rate of appreciation also varies across countries. For LICs to borrow at the same expected cost as conventional debt, either the amount of transferred resources or their concessionality would have to be adjusted to individual countries' expected real appreciations.[22]

The fact that the baseline trend of GDP (or the baseline exchange rate) differs across countries and must be agreed upon by the contracting parties clearly raises incentive issues and adverse selection problems. On the one hand, the borrower country would have an obvious incentive to claim that its growth prospects are strong in order to set the highest possible baseline trend for GDP (or the exchange rate) and thus reduce future debt payments. On the other hand, the MDB would want to ensure itself an expected level of reflows comparable to those on conventional loans. This conflict of interests may give rise to lengthy negotiations and an agreement could be difficult to reach. Further complications arise if the borrower has private information about its future growth prospects. If a 'too low' baseline GDP trend were set, only countries with weak growth potentials would self-select to receive indexed loans.[23]

This discussion suggests that contingent debt contracts have to be carefully designed and more research is needed to better understand their effects before a program of GDP indexation or local currency lending could ever be introduced.

8. Conclusion

In this paper we have evaluated the benefits and costs of indexing MDB loans to variables related to the LICs' ability to pay, and thus whether a reform of multilateral lending is feasible and economically justified. The analysis covers 40 IDA countries over the period from 1990 to 2010, and focuses on three types of debt: GDP-indexed loans; export-indexed loans; inflation-indexed loans denominated in local currency.

Portfolio risk analysis points to a weak correlation of the returns on loans indexed to GDP or

[22] The contract may specify lower amortization payments according to the evolution of the expected (baseline) exchange rate.

[23] It is worth noting that negotiations about the baseline trend of GDP, or the borrower's choice between indexed and conventional loans may reveal private information by the borrower about the growth prospects of its economy and thus on the effectiveness of aid. This opens up the possibility for multilateral lenders to offer a menu of different loan contracts as a way of screening the different types of borrowers along the lines suggested by Froot et al. (1989).

denominated in local currency with OECD growth and thus with the fiscal resources of multilateral lenders. It also suggests that individual country risk can be easily diversified away in the MDBs' portfolio. This conclusion holds true whether beta coefficients from a CAPM are compared across countries or the volatility of the MDBs' portfolio is compared to the risk of individual countries' loans. Hence, there are ample opportunities for risk-sharing among IDA countries, making it feasible for multilateral lenders to provide loans either indexed to GDP or denominated in local currencies. Such loans could be extended at current interest rates, since the estimated risk premium is less than one percent and the additional return-risk for multilateral lenders would be more than offset by a lower frequency of debt crises. By contrast, export indexation does not appear a viable option since the positive correlation of IDA countries' exports with the GDP of OECD countries would expose multilateral lenders to significant systematic risk.

The role of indexed loans in reducing IDA countries' vulnerability to adverse macroeconomic shocks clearly emerges from the estimation of a model where indexed debt may help to stabilize the debt ratio and reduce the likelihood of a debt crisis. We find strong evidence in favor of inflation-indexed local-currency loans and some support for GDP indexation. Lending in the borrower's currency helps to stabilize the debt ratio against unanticipated movements in the real exchange rate which are a main cause of debt vulnerability. We also find supportive evidence for GDP-indexed loans but, while such instruments provide valuable insurance to a majority of IDA borrowers, they benefit a fewer number of countries than domestic currency loans. A main lesson from our analysis is that a 'one size fits all solution' does not exist to the problem of stabilizing the debt ratio. This may hinder a reform of multilateral lending which appeals to all IDA countries.

Both GDP-indexed loans and inflation-indexed loans denominated in local currency raise incentive issues and heighten the risk of moral hazard. Local currency denomination may lead to greater laxity in monetary policy whereas GDP indexation may favor misreporting. The low quality of LICs' inflation and GDP data is also a problem. The relevance of these problems and the ensuing costs for multilateral lenders are difficult to assess but the benefits of contingent debt appear, in both cases, to exceed costs. If anything, the case for relying on the disciplinary effects of conventional debt is weak, as shown by the long history of debt defaults, debt restructuring and relief.

Multilateral loans are immune from the pricing difficulties and liquidity problems of indexed bonds but are sensitive to the contract design. In the case of GDP-indexed loans, the MDB and the borrower country must agree upon a country-specific baseline trend of GDP, since amortization payments must be linked to deviations of realized GDP from its baseline trend for such loans to be attractive to countries with a high growth potential. This clearly raises incentive issues and adverse selection problems. We expect the borrowing country to strongly bargain over a 'high' baseline trend in order to reduce future payments. Although a baseline exchange rate does not need to be specified for lending in local currency, the expected cost of debt service is likely to increase because the real exchange rate of LICs tends to appreciate in the long run, and at different rates in different countries. This implies that the concessionality of local currency loans, i.e. their amortization payments, would have to be adjusted to individual countries' expected real appreciations for such loans to be preferred to conventional loans. Both in the case of GDP indexation and domestic currency lending a conflict of interests is likely to arise which makes an agreement difficult to reach. This poses a serious obstacle to a reform of multilateral lending.

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Tables and Figures

Table 1: Volatility of Growth Rates GDP, Exports and Real Exchange Rates

	IDA countries Average	OECD countries Average	IDA countries		IDA countries Corr.	IDA countries Rank Corr.
	Stand Dev	Stand. Dev.	Min	Max	with Debt	with Debt
GDP Growth	5.4	2.9	0.8	31.8	0.53	0.27
	(3.8)	(-2.3)				
Exports Growth	20.8	6.7	7.2	49.6	0.21	0.04
	(19.2)	(6.3)				
RER Dep.	103.5	9.3	2.2	4512.1	0.23	0.55
Without outliers	17.6	9.3	2.2	74.3	0.49	0.49
	(16.7)	(9.3)				
GDP Nom. Growth	14.9	10.7	4.2	30.1		
	(14.4)	(10.6)				

Note: Country averages of standard deviations. Median in parenthesis.

Table 2: Average Growth Rates: 1990-2000 versus 2000-2010

	GDP growth		Export growth		RER depreciation	
	1990-00	2000-10	1990-00	2000-10	1990-00	2000-10
Angola	0.8	11.2	7.4	20.2	1.9	-12.3
Bangladesh	4.8	5.8	13.3	11.6	0.2	-1.9
Benin	4.8	4.0	3.8	11.8	2.6	-7.1
Burundi	-2.0	3.1	-5.1	13.0	2.7	-5.2
Cambodia	7.4	8.0	24.6	14.2	1.9	-3.6
Cameroon	1.4	3.3	0.6	7.8	2.3	-4.8
Congo, Rep.	1.4	4.6	5.9	12.9	0.0	-8.9
Cote d'Ivoire	2.3	1.1	2.2	12.2	2.6	-6.9
Djibouti	-1.8	3.9	-0.1	8.1	-3.9	-3.3
Ethiopia	2.8	8.4	5.2	16.7	6.4	-5.0
Ghana	4.3	5.8	9.5	14.5	5.7	-13.6
Guinea	4.1	2.8	-1.2	7.6	2.4	-0.9
Guyana	4.9	2.4	4.1	5.4	-1.1	-9.4
Haiti	-0.2	0.1	4.7	4.7	-0.8	-6.1
Honduras	3.3	4.1	14.1	5.8	-5.4	-3.8
Kenya	1.9	4.1	2.2	12.4	-2.1	-5.2
Kyrgyz Republic	-4.0	4.0	7.4	15.7	2.6	-8.6
Lao PDR	6.2	7.2	17.3	16.1	-1.0	-7.7
Lesotho	4.0	10.8	10.4	12.8	0.7	-0.3
Madagascar	1.7	2.6	9.7	2.3	-0.6	-5.7
Malawi	3.4	4.6	-0.1	13.8	4.1	-6.4
Maldives	8.2	7.4	9.8	7.6	-1.0	-4.1
Mali	4.0	5.5	4.4	14.2	3.9	-8.3
Moldova	-9.8	5.1	-0.3	13.6	-0.1	-10.6
Mongolia	0.0	6.5	2.2	18.7	7.8	-11.3
Mozambique	5.5	7.9	11.6	15.8	-0.1	-0.5
Nepal	5.0	3.9	11.7	2.1	0.7	-5.9
Nicaragua	3.4	3.4	10.9	12.7	-10.8	-1.8
Niger	1.8	4.5	-5.0	14.6	4.9	-7.1
Nigeria	2.8	6.4	3.7	13.9	-2.1	-8.5
Rwanda	0.4	7.6	-1.1	16.9	4.3	-4.6
Senegal	3.1	4.1	-1.1	10.1	4.9	-6.4
Sierra Leone	-4.6	9.5	-12.5	22.6	-4.6	-1.9
Solomon Islands	2.5	3.5	2.4	10.6	-1.2	-1.0
Sudan	5.8	6.3	13.9	20.3	5.5	-10.5
Tanzania	3.0	7.0	9.7	16.7	-5.9	-1.5
Togo	2.2	2.6	-4.4	12.2	4.1	-6.3
Uganda	6.5	7.4	14.1	18.0	2.7	-3.0
Vanuatu	3.4	3.1	7.8	4.9	-2.1	-6.5
Yemen, Rep.	5.5	3.9	10.4	8.8	-1.3	-7.9

Table 3: Optimal Shares of Indexed Debt - 5-year Horizon

Countries	Debt Ratio	Risk	Relative Variances V(x)/V(g)	Relative Variances V(e-p)/V(g)	Optimal Shares γ^*	Optimal Shares x^*	Optimal Shares h^*
Angola	22		21.7	6.6	9.0	2.0	3.6
Bangladesh	25	L	228.9	33.4	4.4	0.2	0.9
Benin	18	L	249.1	339.3	-6.7	0.5	0.6
Burundi	33	H	82.7	12.0	-1.2	0.2	1.0
Cambodia	42	M	3.7	3.5	3.3	1.5	1.7
Cameroon	13	L	156.2	206.4	6.4	0.6	1.0
Congo, Rep.	32	L	35.5	58.9	10.0	1.7	1.3
Cote d'Ivoire	50	D	11.5	14.2	0.7	0.8	0.7
Djibouti	72	H	22.6	0.7	-0.7	-0.1	-1.6
Ethiopia	24	L	11.4	30.8	1.3	0.6	0.4
Ghana	27	M	3.3	118.4	15.5	5.3	1.5
Guinea	65	D	117.1	34.2	0.3	0.2	0.9
Guyana	61	M	2.7	15.3	5.6	3.1	1.3
Haiti	7	H	198.2	186.4	50.6	4.4	5.5
Honduras	27	L	6.5	3.1	-1.6	0.4	1.7
Kenya	27	L	182.1	149.9	0.9	-1.4	1.6
Kyrgyz Republic	86	M	13.8	7.2	4.1	1.0	1.6
Lao PDR	76	H	238.4	163.6	10.3	0.6	0.9
Lesotho	34	M	3.6	1.4	-0.2	0.4	2.0
Madagascar	26	L	5.4	7.0	-0.1	0	1.1
Malawi	18	M	5.6	18.1	5.6	0.5	-0.2
Maldives	64	H	6.1	0.6	3.0	1.3	1.7
Mali	25	M	30.3	22.6	6.3	1.0	1.5
Moldova	79	L	11.9	3.3	2.1	0.6	1.2
Mongolia	39	L	24.1	8.8	6.5	1.7	1.5
Mozambique	43	L	40.1	35.6	-3.6	-0.4	1.4
Nepal	29	M	51.0	12.7	3.5	0.5	0.5
Nicaragua	73	M	28.0	121.1	2.5	1.5	1.2
Niger	20	M	71.1	50.0	5.8	0.8	0.9
Nigeria	4	L	69.4	35.0	28.6	5.9	7.5
Rwanda	14	M	2.8	0.6	5.7	2.6	4.8
Senegal	28	L	28.7	90.2	7.7	1.2	0.8
Sierra Leone	41	M	18.9	0.9	0.7	0.1	1.3
Solomon Islands	32	M	16.2	1.1	2.8	0.5	-0.1
Sudan	35	D	125.7	56.5	-1.4	0.7	1.0
Tanzania	38	L	194.2	75.1	4.7	0	0.8
Togo	55	M	9.7	12.0	-0.3	0.9	1.0
Uganda	18	L	39.6	13.2	3.1	0.6	1.5
Vanuatu	21	L	8.3	10.9	-4.1	-1.1	-2.4
Yemen, Rep.	24	H	58.1	110.8	-0.5	1.3	0.6

Table 4: Optimal Shares of Indexed Debt - 10-year Horizon

Countries	Debt Ratio	Risk	Relative Variances V(x)/V(g)	Relative Variances V(e-p)/V(g)	Optimal Shares γ^*	Optimal Shares x^*	Optimal Shares h^*
Angola	22		12.4	3.1	5.0	1.6	3.2
Bangladesh	25	L	138.1	17.9	2.8	0.2	0.9
Benin	18	L	55.4	16.4	1.8	-0.8	1.4
Burundi	33	H	14.7	3.7	-2.8	0.2	1.5
Cambodia	42	M	2.6	3.0	3.3	2.1	1.9
Cameroon	13	L	123.0	226.2	8.9	0.2	0.9
Congo, Rep.	32	L	35.9	59.1	5.8	1.0	0.7
Cote d’Ivoire	50	D	4.3	9.2	-0.5	0.8	0.5
Djibouti	72	H	14.1	0.6	-0.7	-0.3	-1.5
Ethiopia	24	L	10.0	134.8	1.5	0.6	0.3
Ghana	27	M	0.03	100.8	14.2	78.5	1.4
Guinea	65	D	1826.8	1293.3	12.0	0.7	0.9
Guyana	61	M	2.0	13.3	5.0	3.6	1.0
Haiti	7	H	191.8	203.8	82.2	5.9	5.8
Honduras	27	L	3.3	1.7	-2.3	1.4	2.2
Kenya	27	L	132.9	201.3	-12.3	-2.2	2.1
Kyrgyz Republic	86	M	13.8	7.2	4.1	1.0	1.6
Lao PDR	76	H	234.6	315.4	8.7	0.5	0.9
Lesotho	34	M	2.1	1.8	1.4	0.9	4.8
Madagascar	26	L	5.5	7.5	-0.2	-0.1	1.2
Malawi	18	M	4.6	17.8	5.3	0.1	-0.3
Maldives	64	H	6.1	0.6	3.0	1.3	1.7
Mali	25	M	68.3	24.9	0.9	0.8	1.6
Moldova	79	L	13.3	3.6	2.2	0.6	1.2
Mongolia	39	L	20.0	21.7	-0.6	2.6	-3.9
Mozambique	43	L	40.9	38.0	-1.3	-0.2	1.5
Nepal	29	M	68.4	14.3	4.7	0.6	0.6
Nicaragua	73	M	24.2	70.7	2.6	0.9	1.2
Niger	20	M	136.4	86.5	7.0	0.6	0.8
Nigeria	4	L	45.5	22.3	33.0	5.6	7.6
Rwanda	14	M	3.4	0.7	5.6	2.4	5.0
Senegal	28	L	32.6	97.0	7.3	1.1	0.8
Sierra Leone	41	M	15.9	0.6	1.1	0.2	1.3
Solomon Islands	32	M	12.1	0.9	2.7	0.6	0.8
Sudan	35	D	104.2	65.3	-1.7	0.3	0.2
Tanzania	38	L	328.3	52.0	8.5	0.1	0.5
Togo	55	M	0.9	1.2	1.3	1.2	0.9
Uganda	18	L	31.6	9.2	4.1	0.4	1.3
Vanuatu	21	L	9.2	26.3	-13.2	-4.3	-2.8
Yemen, Rep.	24	H	37.7	97.2	-2.9	0.5	0.2

Table 5: Risk Diversification - Portfolio Returns and Volatilities

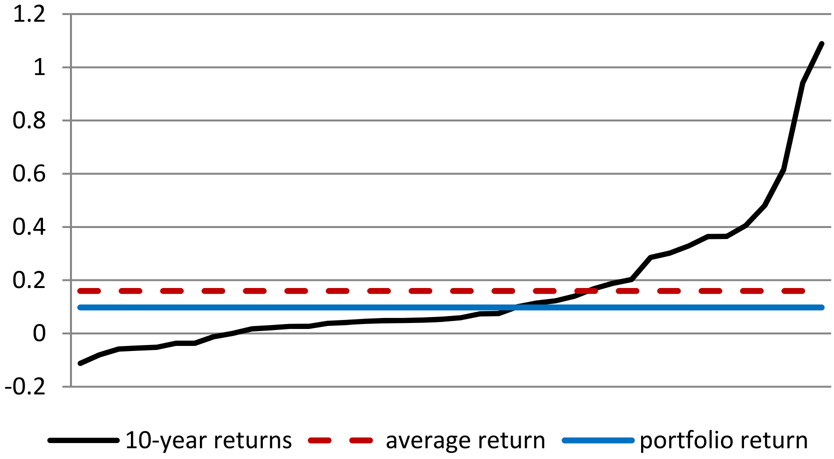
	GDP-indexed	Export-indexed	Domestic-currency Inflation-indexed
		Returns – percent	
Portfolio return	9.7	31.1	39.6
Individual loan returns			
Min	-11.2	-37.1	-35.2
Max	108.9	369.4	144.6
Mean	16.0	46.9	42.7
st.dev.	25.5	67.4	39.3
25° perc.	1.8	6.7	16.0
50° perc	5.6	41.8	39.5
75° perc	26.5	58.4	65.4
90° perc	47.3	121.3	91.2
		Volatility - percent	
Portfolio risk	1.0	8.8	4.6
Individual loan risk			
Min	0.5	5.6	2.3
Max	27.8	33.1	21.7
Mean	3.4	15.1	9.6
st.dev.	4.3	6.7	4.6
10° perc.	0.7	8.4	3.7
25° perc.	1.4	9.6	6.6
50° perc	2.5	13.3	8.8
75° perc	3.7	19.7	12.2
90° perc.	6.8	26.3	17.8

Table 6: Beta Coefficients of Indexed Debt

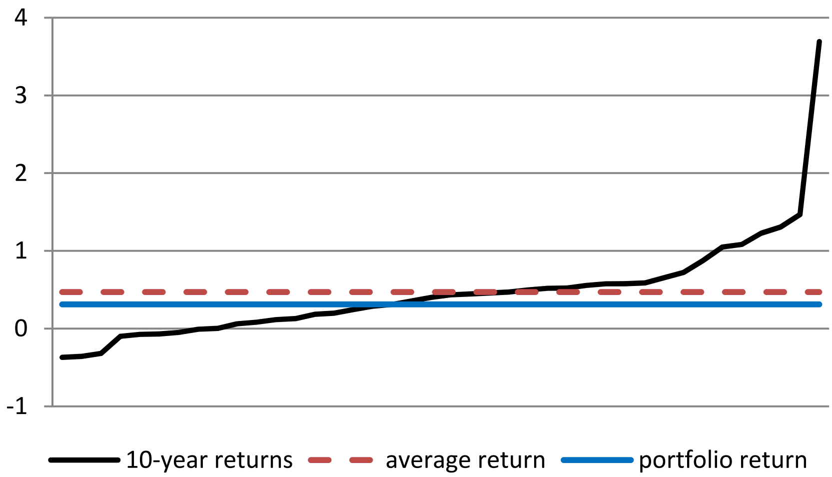
Countries	5-year horizon GDP	5-year horizon Export	5-year horizon Dom-Cur	10-year horizon GDP	10-year horizon Export	10-year horizon Dom-Curr
Angola	0.3	6.7	1.5	0.4	8.5	2.9
Bangladesh	0.3	4.2	-0.2	0.3	4.0	-0.4
Benin	0.6	-5.4	-3.9	0.2	-2.8	-2.0
Burundi	0.1	2.9	-2.1	-0.4	-3.1	-4.7
Cambodia	1.5	2.2	-0.2	1.6	3.1	-0.7
Cameroon	0.3	2.6	-0.5	0.5	2.6	-0.8
Congo, Rep.	-1.2	5.3	6.9	-1.9	0.2	0.7
Cote d'Ivoire	-1.4	-1.3	1.6	0.5	-4.5	-2.7
Djibouti	0.1	-2.5	0	0.2	-1.7	0.3
Ethiopia	-0.9	2.2	-1.7	0.1	7.3	-1.4
Ghana	-0.2	0	-1.7	0.1	1.3	-2.6
Guinea	0.5	2.5	-0.9	0.5	0.4	-0.9
Guyana	0.5	1.2	1.1	0	0.4	3.1
Haiti	0.2	3.6	-1.6	0.2	2.8	-2.7
Honduras	0.1	4.4	0.3	0	4.7	0.6
Kenya	0.5	4.3	0.1	0.6	11.9	-1.8
Kyrgyz Rep.	-0.1	0.1	-0.3	-0.2	-0.5	-0.6
Lao PDR	-0.2	-0.4	-3.5	-0.2	2.8	-1.4
Lesotho	-0.6	4.1	-1.5	-0.3	6.1	-1.7
Madagascar	2.1	4.4	-6.5	2.1	4.4	-3.9
Malawi	-1.6	-6.4	-1.4	-1.8	-6.4	-1.3
Maldives	0.7	0.3	-1.0	1.6	3.1	-0.9
Mali	1.2	2.8	0	1.5	3.2	-0.5
Moldova	1.2	2.7	-0.2	1.1	2.3	0.1
Mongolia	0.9	6.9	2.9	0.8	8.2	3.4
Mozambique	-0.3	3.6	1.1	-0.6	4.4	1.7
Nepal	0.3	-0.7	0	0.2	-3.0	-0.8
Nicaragua	0.7	-0.9	-1.1	0.1	-3.5	-1.3
Niger	0.6	-2.0	-0.8	1.0	0	0.4
Nigeria	-0.4	6.1	4.0	-0.9	3.5	2.8
Rwanda	1.8	0.9	-1.2	-1.7	-12.7	-11.7
Senegal	0.4	0.3	0.9	0.3	-0.3	1.5
Sierra Leone	-2.1	-2.9	1.2	-2.2	1.4	1.7
Solomon Isl.	0.5	0.9	-0.4	1.8	9.7	-0.4
Sudan	-0.3	8.3	2.3	-0.1	5.7	-4.0
Tanzania	0.1	0.5	-0.1	0	0.7	-0.6
Togo	-0.8	0.4	-0.6	-1.5	2.7	1.0
Uganda	-0.4	-1.7	-0.4	0.1	1.4	1.6
Vanuatu	3.1	7.6	-3.4	3.5	8.6	-2.5
Yemen, Rep.	0	6.0	4.7	0.6	2.9	2.1
N.Obs Beta > 0.5	11	24	11	11	26	12
Mean	0.21	1.85	-0.16	0.20	2.00	-0.71
Median	0.27	2.23	-0.27	0.19	2.61	-0.63
Portfolio Mul. Debt	0.14	2.70	0.13	0.19	2.83	-0.54
Risk Premium (percent)	0.67	8.80	0.62	0.89	9.09	-3.03
Portfolio Disbursement	0.04	2.40	0.31	0.07	2.74	-0.14
Risk Premium (percent)	0.19	8.09	1.43	0.34	8.89	-0.71

Figure 1: Unexpected 10-Year returns

GDP-indexed loans



Export-indexed loans



Domestic Currency loans

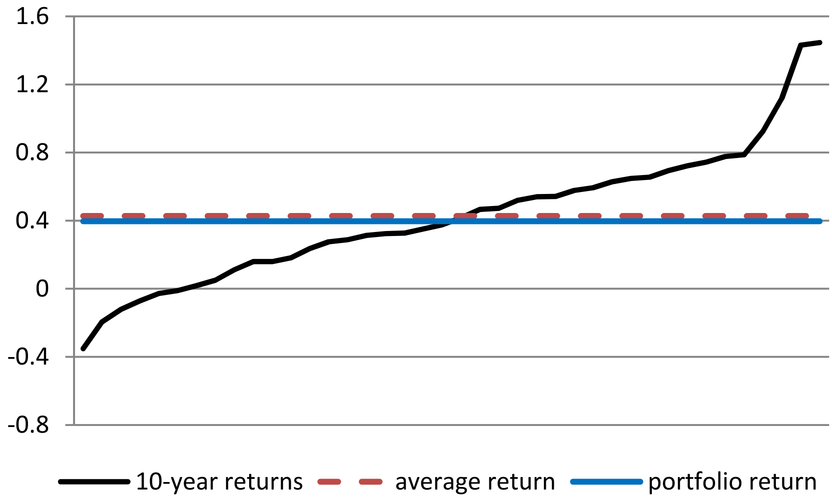
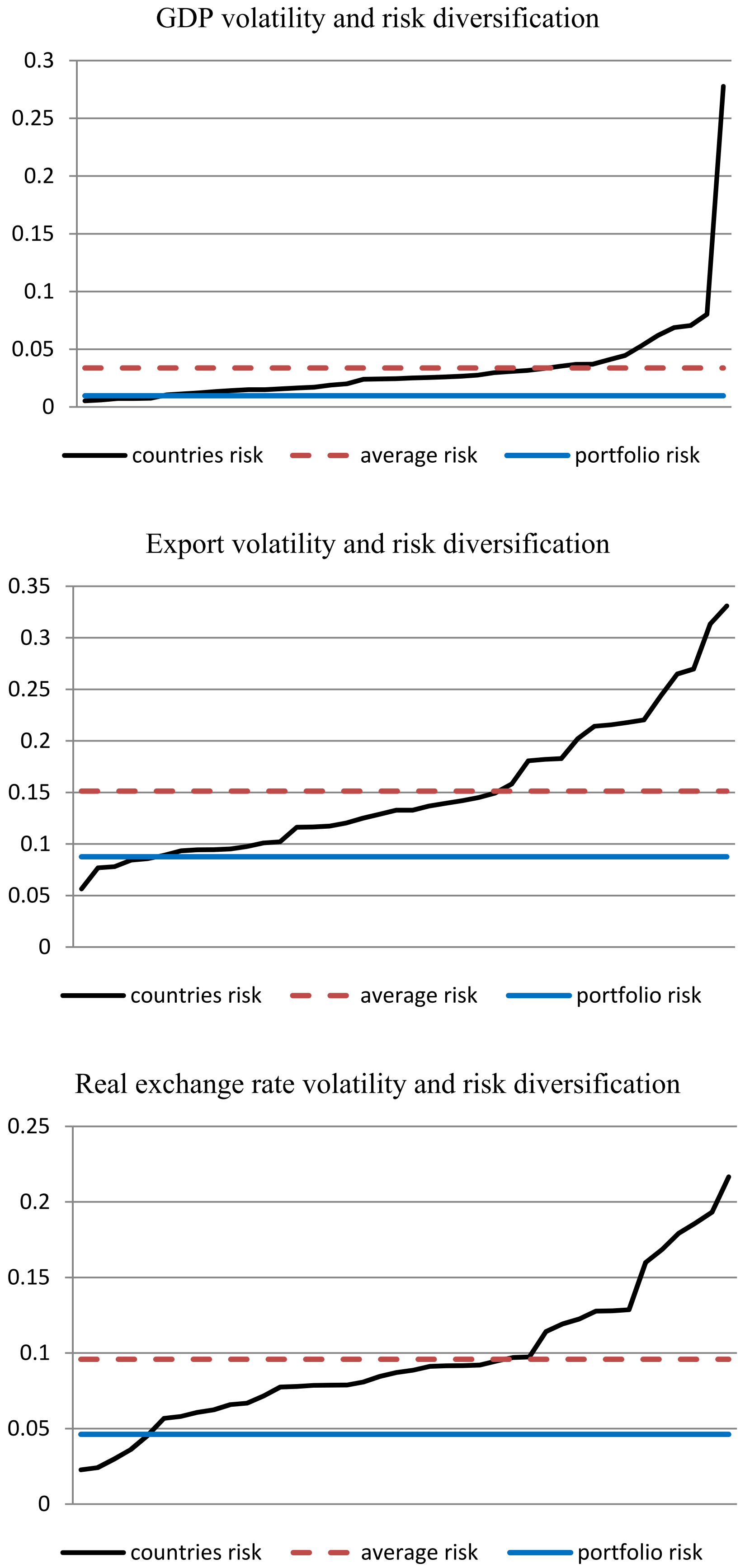


Figure 2: Risk diversification opportunities



Technical Appendix: The econometric model

We assume that the Data Generating Process for the macroeconomic variables follows a Vector Autoregressive (VAR) process of the form

$$Y_t = A_1 Y_{t-1} + \ldots + A_p Y_{t-p} + B_0 X_t + \ldots + B_p X_{t-p} + \phi \mu_t + \varepsilon_t \quad (24)$$

where Y_t is the $k \times 1$ vector of dependent variables, X_t is the $k \times 1$ vector of exogenous variables, μ_t contains deterministic components, and $\left(A_1, \ldots, A_p, B_0, \ldots, B_p, \phi\right)$ are matrices of parameters to be estimated. Finally, ε_t is a vector of error terms with the usual assumption $\varepsilon_t \sim N\left(0, \Sigma\right)$.
Given the presence of unmodeled exogenous variables, this kind of model is generally referred to VARX or dynamic simultaneous equations model (SEM)[24].
However, if the aim of the analysis is to forecast the dependent variables, Y_t, and the future paths of the unmodeled variables are unknown to the forecaster, a simple and practical assumption is to model the exogenous variables X_t as a VAR(q) process

$$X_t = C_1 X_{t-1} + \ldots + C_q X_{t-q} + \varphi \nu_t + \nu_t . \quad (25)$$

The two reduced form models for Y_t and X_t can thus be considered together in one overall model of the form

$$\begin{bmatrix} I_k & -B_0 \\ 0 & I_m \end{bmatrix} \begin{pmatrix} Y_t \\ X_t \end{pmatrix} = \begin{bmatrix} A_1 & -B_1 \\ 0 & C_1 \end{bmatrix} \begin{pmatrix} Y_{t-1} \\ X_{t-1} \end{pmatrix} + \ldots + \begin{bmatrix} A_p & -B_p \\ 0 & C_p \end{bmatrix} \begin{pmatrix} Y_{t-p} \\ X_{t-p} \end{pmatrix} + \begin{bmatrix} \phi & 0 \\ 0 & \varphi \end{bmatrix} \begin{pmatrix} \mu_t \\ \nu_t \end{pmatrix} + \begin{pmatrix} \varepsilon_t \\ \nu_t \end{pmatrix} \quad (26)$$

where it is assumed, without loss of generality, that $p \geq q$ and, as a consequence, $C_j = 0$ for $j > q$. Premultiplying by

$$\begin{bmatrix} I_k & -B_0 \\ 0 & I_m \end{bmatrix}^{-1} = \begin{bmatrix} I_k & B_0 \\ 0 & I_m \end{bmatrix} \quad (27)$$

gives a standard reduced form VAR(p) for $Z_t = \left(Y_t^{'}, X_t^{'}\right)^{'}$ as

$$Z_t = \Pi_1 Z_{t-1} + \ldots + \Pi_p Z_{t-p} + \eta D_t + u_t \quad (28)$$

for which the discussion of forecasting VAR(p) processes applies. More precisely, the Wold moving average decomposition for stable VAR(p) processes is defined as

$$Z_t = \Phi_0 \left(u_t + \eta D_t\right) + \Phi_1 \left(u_{t-1} + \eta D_{t-1}\right) + \Phi_2 \left(u_{t-2} + \eta D_{t-2}\right) + \ldots, \quad (29)$$

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24 For a detailed discussion of these models see Lütkepohl (2005), Chapter 10.

with $\Phi_0 = I_{k+m}$ and Φ_s computed recursively according to

$$\Phi_s = \sum_{j=1}^{s} \Phi_{s-j} \Pi_j \qquad \text{for} \quad s = 1, 2, \ldots \tag{30}$$

whereby $\Pi_j = 0$ for $j > p$. Given the two representations for the stochastic process Z_t, forecasts for horizons $h \geq 1$ can be generated recursively according to

$$Z_{t+h|t} = \Pi_1 Z_{t+h-1} + \ldots + \Pi_p Z_{t+h-p} + \eta D_{t+h} \tag{31}$$

where $Z_{t+j|t} = Z_{t+j}$ for $j \leq 0$, while the forecast error covariance or MSE matrix can be obtained as

$$\Sigma_z(h) = \sum_{i=1}^{h-1} \Phi_i \Sigma_u \Phi_i' \tag{32}$$

where Σ_u is the covariance matrix of the error term u_t.
The first $(k \times k)$ block elements of $\Sigma_z(h)$ are the variances and covariances of the unanticipated components of the endogenous variables $Y_{t+h|t}$ at different horizons, h.

Table A1: Covariance-Variance Ratios - 5-years ahead unexpected components

	$\frac{Cov(g_{t+1}\tilde{e}_{t+1})}{Var(g_{t+1})}$	$\frac{Cov(g_{t+1}nx_{t+1})}{Var(g_{t+1})}$	$\frac{Cov(v_{t+1}g_{t+1})}{Var(v_{t+1})}$	$\frac{Cov(v_{t+1}\tilde{e}_{t+1})}{Var(v_{t+1})}$	$\frac{Cov(v_{t+1}nx_{t+1})}{Var(v_{t+1})}$	$\frac{Cov(\tilde{e}_{t+1}g_{t+1})}{Var(\tilde{e}_{t+1})}$	$\frac{Cov(\tilde{e}_{t+1}nx_{t+1})}{Var(\tilde{e}_{t+1})}$	$\frac{Var(v_{t+1})}{Var(g_{t+1})}$	$\frac{Var(\tilde{e}_{t+1})}{Var(g_{t+1})}$
Angola	-2.411	1.220	0.209	-0.542	0.279	-0.364	-0.494	21.7	6.6
Bangladesh	-3.135	0.071	0.044	-0.242	-0.022	-0.094	0.049	228.9	33.4
Benin	4.011	-0.685	-0.014	-0.803	-0.055	0.012	0.064	249.1	339.3
Burundi	0.251	-0.652	0.035	-0.308	-0.036	0.021	0.006	82.7	12.0
Cambodia	-1.771	0.202	0.423	-0.856	0.077	-0.508	-0.100	3.7	3.5
Cameroon	-1.939	0.460	0.032	-0.532	0.003	-0.009	0.006	156.2	206.4
Congo, Rep.	-7.621	0.450	0.167	-1.283	0.076	-0.129	-0.057	35.5	58.9
Cote d'Ivoire	-0.077	-0.169	0.038	-0.582	0.102	-0.005	0.148	11.5	14.2
Djibouti	-0.081	-1.285	0.038	-0.015	-0.077	-0.117	1.983	22.6	0.7
Ethiopia	-0.947	-0.159	0.179	-0.875	-0.112	-0.031	0.147	11.4	30.8
Ghana	-10.685	1.017	0.267	-3.314	0.467	-0.090	-0.097	3.3	118.4
Guinea	1.758	0.667	-0.037	-0.340	-0.057	0.051	0.047	117.1	34.2
Guyana	-1.139	2.134	0.438	-0.850	1.098	-0.074	-0.121	2.7	15.3
Haiti	-8.998	2.977	0.061	-0.835	0.259	-0.048	-0.329	198.2	186.4
Honduras	0.692	-0.525	-0.006	-0.004	0.110	0.224	-0.238	6.5	3.1
Kenya	-2.463	-0.684	0.023	0.151	-0.336	-0.016	-0.160	182.1	149.9
Kyrgyz Rep.	-1.532	1.354	0.202	-0.471	0.282	-0.212	-0.347	13.8	7.2
Lao PDR	-11.840	-1.928	0.053	-0.649	-0.093	-0.072	0.150	238.4	163.6
Lesotho	0.450	-0.265	0.289	-0.056	0.028	0.314	-0.462	3.6	1.4
Madagascar	1.332	0.055	-0.067	0.274	0.087	0.191	-0.086	5.4	7.0
Malawi	1.911	1.175	0.122	0.351	0.129	0.106	0.189	5.6	18.1
Maldives	-0.034	1.284	0.386	-0.085	0.545	-0.058	-0.417	6.1	0.6
Mali	-3.087	0.544	0.029	-0.456	0.120	-0.137	-0.102	30.3	22.6
Moldova	-1.611	-0.381	0.279	-0.505	-0.116	-0.493	0.197	11.9	3.3
Mongolia	-2.313	1.274	0.193	-0.490	0.385	-0.264	-0.099	24.1	8.8
Mozambique	3.310	-0.573	0.053	0.491	0.020	0.093	-0.227	40.1	35.6
Nepal	-1.182	0.370	0.130	-0.270	0.035	-0.093	0.157	51.0	12.7
Nicaragua	-3.899	-1.774	0.011	-1.181	0.254	-0.032	-0.138	28.0	121.1
Niger	-5.414	-0.127	0.098	-0.816	-0.030	-0.108	0.033	71.1	50.0
Nigeria	-4.130	0.956	0.090	-0.641	0.210	-0.118	-0.261	69.	35.0
Rwanda	-0.329	0.618	0.435	-0.320	0.263	-0.529	-0.456	2.8	0.6
Senegal	-8.925	-0.642	0.164	-1.601	-0.155	-0.099	0.072	28.7	90.2
Sierra Leone	0.417	0.042	0.179	0.049	-0.002	0.464	-0.321	18.9	0.9
Solomon Isl.	-0.044	0.555	0.190	-0.024	0.086	-0.040	0.356	16.2	1.1
Sudan	4.677	0.816	0.018	-0.245	0.146	0.083	-0.031	125.7	56.5
Tanzania	-1.397	0.876	0.033	-0.195	-0.093	-0.019	0.078	194.2	75.1
Togo	1.335	0.024	0.070	-0.781	0.039	0.111	-0.037	9.7	12.0
Uganda	-1.157	0.171	0.046	-0.406	0.019	-0.088	-0.078	39.6	13.2
Vanuatu	-1.901	-1.479	0.302	-0.699	-0.437	-0.174	0.765	8.3	10.9
Yemen, Rep.	4.515	0.734	-0.056	-0.360	0.229	0.041	0.094	58.1	110.8

Note: To save space we use the definition $\tilde{e}_{t+1} = e_{t+1} - \pi_{t+1}$

Table A2: Covariance-Variance Ratios - 10-years ahead unexpected components

	$\frac{Cov(g_{t+1}\tilde{e}_{t+1})}{Var(g_{t+1})}$	$\frac{Cov(g_{t+1}nx_{t+1})}{Var(g_{t+1})}$	$\frac{Cov(v_{t+1}g_{t+1})}{Var(v_{t+1})}$	$\frac{Cov(v_{t+1}\tilde{e}_{t+1})}{Var(v_{t+1})}$	$\frac{Cov(v_{t+1}nx_{t+1})}{Var(v_{t+1})}$	$\frac{Cov(\tilde{e}_{t+1}g_{t+1})}{Var(\tilde{e}_{t+1})}$	$\frac{Cov(\tilde{e}_{t+1}nx_{t+1})}{Var(\tilde{e}_{t+1})}$	$\frac{Var(v_{t+1})}{Var(g_{t+1})}$	$\frac{Var(\tilde{e}_{t+1})}{Var(g_{t+1})}$
Angola	-1.650	0.527	0.277	-0.494	0.175	-0.536	-0.369	12.4	3.1
Bangladesh	-2.485	-0.173	0.063	-0.264	-0.025	-0.139	0.056	138.1	17.9
Benin	-0.715	0.015	-0.026	0.250	-0.102	-0.043	-0.073	55.4	16.4
Burundi	0.989	-0.924	0.065	-0.314	-0.063	0.267	-0.251	14.7	3.7
Cambodia	-1.743	0.233	0.621	-1.082	0.145	-0.574	-0.134	2.6	3.0
Cameroon	-5.156	0.360	0.041	-0.497	-0.041	-0.023	0.022	123.0	226.2
Congo, Rep.	-7.690	-0.930	0.167	-1.283	-0.155	-0.130	0.121	35.9	59.1
Cote d'Ivoire	1.962	0.233	-0.207	-0.905	0.032	0.213	0.164	4.3	9.2
Djibouti	-0.064	-1.279	0.068	-0.053	-0.317	-0.107	1.842	14.1	0.6
Ethiopia	-1.785	-0.308	0.170	-1.063	-0.151	-0.013	0.184	10.0	134.8
Ghana	-10.038	0.843	5.522	-55.481	4.672	-0.100	-0.084	0.03	100.8
Guinea	-11.308	-0.224	0.007	-0.826	-0.061	-0.009	0.073	1826.8	1293.3
Guyana	-0.307	2.266	0.614	-0.601	1.440	-0.023	0.023	2.0	13.3
Haiti	-14.118	4.916	0.072	-1.014	0.352	-0.069	-0.350	191.8	203.8
Honduras	1.080	-0.598	-0.394	-0.513	0.340	0.626	-0.495	3.3	1.7
Kenya	4.278	-2.422	0.053	0.748	-0.415	0.021	-0.296	132.9	201.3
Kyrgyz Rep.	-1.520	1.344	0.202	-0.469	0.282	-0.211	-0.338	13.8	7.2
Lao PDR	-10.132	-1.832	0.062	-0.551	-0.106	-0.032	0.100	234.6	315.4
Lesotho	-0.048	0.121	0.241	-0.133	0.192	-0.027	-1.271	2.1	1.8
Madagascar	1.344	0.043	-0.076	0.286	0.075	0.180	-0.105	5.5	7.5
Malawi	1.926	1.127	0.065	0.245	0.051	0.108	0.215	4.6	17.8
Maldives	-0.035	1.286	0.385	-0.085	0.547	-0.060	-0.412	6.1	0.6
Mali	-0.373	-0.113	-0.051	-0.443	0.109	-0.015	-0.139	68.3	24.9
Moldova	-1.857	-0.540	0.272	-0.516	-0.149	-0.517	0.283	13.3	3.6
Mongolia	-3.024	-1.818	0.190	-0.264	0.863	-0.139	1.970	20.0	21.7
Mozambique	0.692	-0.672	0.078	0.200	-0.051	0.018	-0.207	40.9	38.0
Nepal	-1.469	0.639	0.116	-0.223	0.073	-0.103	0.137	68.4	14.3
Nicaragua	-2.783	-0.874	0.019	-0.848	0.056	-0.039	-0.122	24.2	70.7
Niger	-8.923	-0.603	0.083	-0.794	-0.056	-0.103	0.070	136.4	86.5
Nigeria	-4.279	1.127	0.137	-0.670	0.196	-0.192	-0.259	45.5	22.3
Rwanda	-0.326	0.608	0.384	-0.287	0.244	-0.438	-0.499	3.4	0.7
Senegal	-9.815	-1.001	0.170	-1.643	-0.194	-0.101	0.097	32.6	97.0
Sierra Leone	0.289	0.153	0.210	0.043	0.028	0.451	-0.316	15.9	0.6
Solomon Isl.	-0.308	0.429	0.237	-0.075	0.090	-0.345	0.161	12.1	0.9
Sudan	8.011	1.856	0.040	0.287	0.200	0.123	0.222	104.2	65.3
Tanzania	-2.606	1.829	0.033	-0.294	-0.070	-0.050	0.206	328.3	52.0
Togo	0.141	0.236	0.146	-0.950	0.064	0.121	-0.024	0.9	1.2
Uganda	-0.848	0.395	0.031	-0.403	-0.013	-0.092	-0.036	31.6	9.2
Vanuatu	-4.743	-4.011	0.320	-1.582	-1.317	-0.180	0.845	9.2	26.3
Yemen, Rep.	5.294	0.327	-0.143	-0.325	0.086	0.054	0.171	37.7	97.2

Note: To save space we use the definition $\tilde{e}_{t+1} = e_{t+1} - \pi_{t+1}$